ASSIGNMENT 7

DUE DATE: NOV 29, 2011

1) Let R be a commutative ring without zero divisors. If $a, b \in R$ and $a^n = b^n$, and $a^m = b^m$ for two relatively prime positive integers m and n, prove that $a = b$.

10pts

2) Find the greatest common divisor of $(3+4i)$ and $(4-3i)$ in the euclidean ring $\mathbb{Z}[i]$.

10pts

3) If U and V are ideals of R, let UV be the set of all elements that can be written as a *finite* set of elements of the form uv where $u \in U$ and $v \in V$. Prove that UV is an ideal of R and that $UV \subset U \cap V$.

10pts

4) Let R be a ring with unit element. Define new operations $\oplus$ by $a \oplus b = a + b + 1$ and $a \cdot b = ab + a + b$ where $a + b$ and ab are the old addition and multiplication operations in the ring R. Prove that $R' = (R, \oplus, \cdot)$ is again a ring, write the zero element and unit elements of R.

10pts

5) Prove that the rings R and R' are isomorphic.

10pts