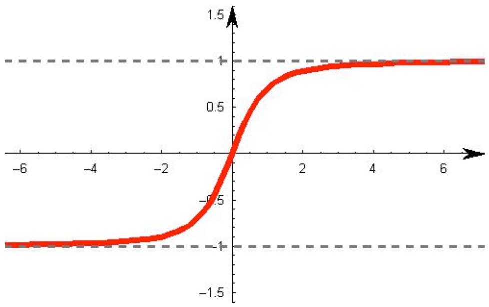
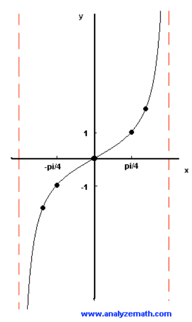
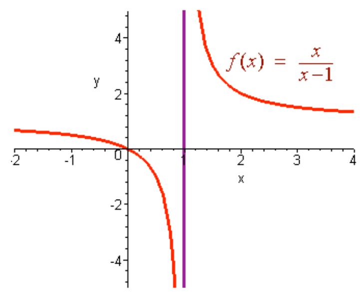
Polynomial Long Division and Finding Vertical and Slant Asymptotes

An **asymptote** is a line that a function approaches but never touches as it stretches to positive and/or negative infinity.







Vertical asymptotes stand parallel to the y- axis. They are obtained by determining the value(s) of x that make the function undefined. When working with rational functions, the denominator is equated to 0 and solved for x to find vertical asymptotes.

Ex 1.

Find the vertical asymptotes:

y= $\frac{4x+1}{3x+3}$

The first thing we need to understand when given a fraction is that the **denominator can never be 0**. Dividing by 0 is undefined. This means that any value of x that causes the devisor to equal 0 is an asymptote. The function cannot have the said value of x and therefore does not pass any points with that particular value. All of the points with this value of x line up to form a perfectly vertical line.

To find the vertical asymptote, the denominator must be equated to zero and solved for x.

3x+3=0
x=-3/3
x= -1

The function has a vertical asymptote at **x = -1**

Ex 2.

$$y = \frac{(7x\ -\ 24)^2}{2x^3 + 3x^2 - 8x - 12}$$

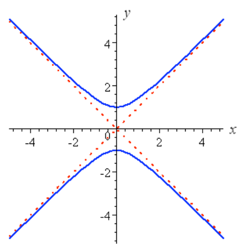
To find the values of x that produces a value of 0 in the denominator, we must first factor since when either of the factors of any function equals zero, the entire function equals zero. Factoring, we get:

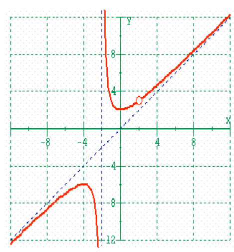
$$y = \frac{(7x-24)^2}{(2x+3)(x^2-4)}$$

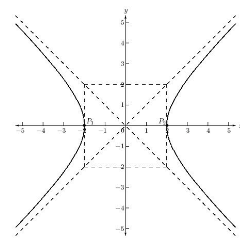
- ⇨ $(2x+3)(x^2-4) = 0$
- ⇨ $2x+3=0$
- ⇨ $2x = -3$
- ⇨ $x = -3/2$

$x^2 - 4 = 0$
$x^2 = 4$
$x = + 2$

Therefore, the function has vertical asymptotes at x = - 3/2, - 2, 2.







Slant asymptotes are lines that are not parallel to the x- or y- axis that a function may approach but will never reach or pass. Slant asymptotes are observed in rational functions where the degree of the leading polynomial in the numerator is one higher than the degree of the polynomial in the denominator. When these polynomials are divided, the quotient will represent a slant asymptote to the function.

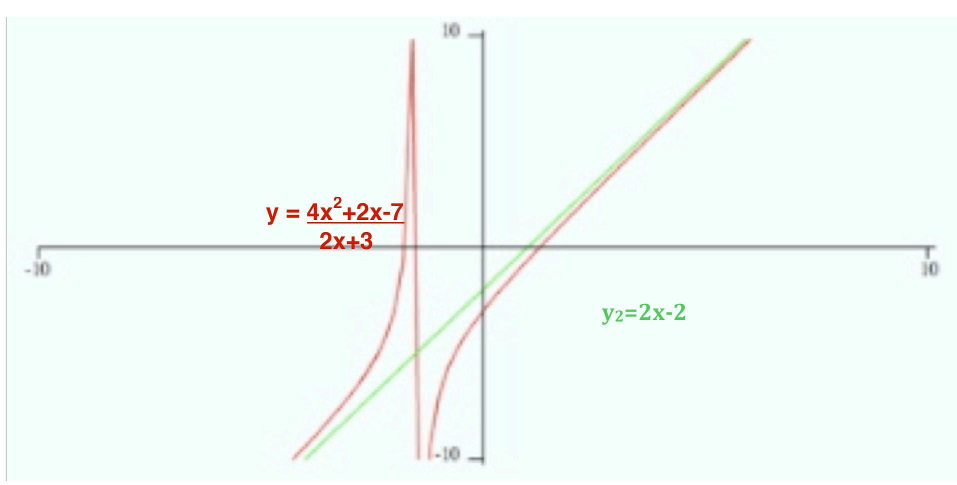
[Ex. 3]

Find the equation for the slant asymptote for the following functions:

a) $y = \dfrac{4x^2+2x-7}{2x+3}$

SOLUTION:
(perform polynomial long division)

$\underline{2x-2\quad\quad}$
$2x+3 \,/\, 4x^2+2x-7$
$\underline{4x^2-6x}$
$-4x-7$
$\underline{-4x-6}$
-1



After doing polynomial long division, we see that the quotient equals:

$$\frac{4x^2+2x-7}{2x+3} = 2x-2 - \frac{1}{4x^2+2x-7}$$

The remainder portion is arbitrary in the equation of the slant asymptote since as x approaches $\pm$ infinity, the value of the remainder approaches zero.

Therefore, **y= 2x-2** represents the slant asymptote for this graph.

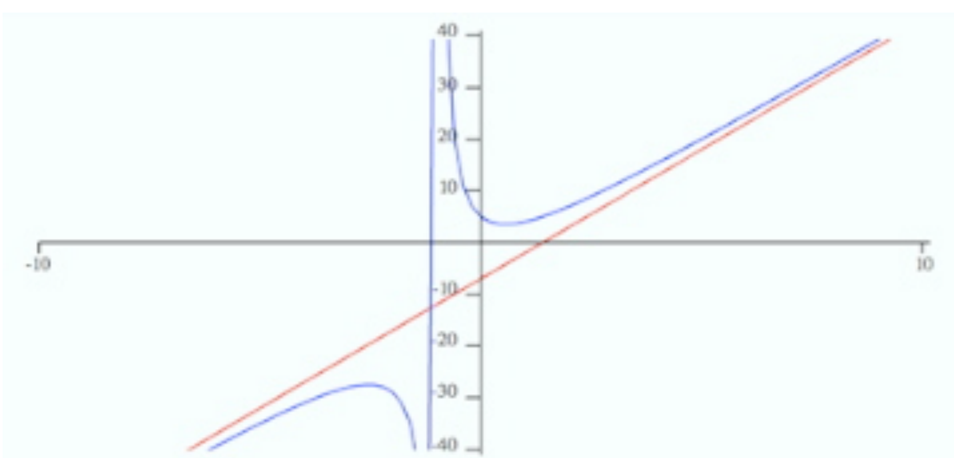
b) $$\mathbf{y = \frac{5x^2-2x+5}{x+1}}$$

(*perform polynomial long division)*

$$\begin{array}{r|l} & \ \ 5x - 7 \\ \hline x+1 & 5x^2 - 2x + 5 \\ & \underline{5x^2 + 5x} \\ & \quad\ \ -7x + 5 \\ & \quad\ \ \underline{-7x - 7} \\ & \qquad\quad 12 \end{array}$$

The equation for the slant asymptote is y = 5x-7

Graphing the asymptote (red) and the original function (blue), we see:



The function seems to come very close to the asymptote but never actually touches this line.