Rotation-Powered Neutron Stars

Spinning magnets in the sky

Are pulsars rotating neutron stars?

- Things to remember:
  - Periods range from 1.6 ms to 8 s.
  - Pulsar periods increase very slowly and don't decrease except for glitches.
  - Pulsars are stable clocks.
- Size: $r < cP < 500$ km so it could be a white dwarf, black hole or neutron star.

Maximal Rotation Frequencies

- Equate the centripetal force to the gravitational force at the surface:

$$\Omega^2 R < \frac{GM}{R^2} \text{ so } \Omega < \left(\frac{GM}{R^3}\right)^{1/2}$$

- Using $\rho \sim 10^8$ g cm^{-3} gives $\Omega \sim 5.3$ Hz or $P \sim 1$ s (a white dwarf can't spin that fast)
- Using $\rho \sim 10^{15}$ g cm^{-3} gives $\Omega \sim 16$ kHz or $P \sim$ 0.4ms (a neutron star can spin fast enough)

Pulsation Frequencies

- The fast pulsation modes of a star are pressure modes, i.e. sound waves.
  - We need to estimate the speed of sound

$$c_s^2 = \frac{dP}{d\rho} \sim \frac{P}{\rho}$$

  - We have an estimate for the density but what about *P*? For a constant density star, the gravitational acceleration is proportional to the distance from the center!

$$P = \int_0^R \frac{GM}{R^2} \rho \frac{r}{R} dr = \frac{GM}{R^2} \rho \frac{R}{2}$$

$$c_s^2 \sim \frac{P}{\rho} = \frac{GM}{2R} \quad \omega = \frac{2\pi c_s}{R} = 2\pi \left(\frac{GM}{2R^3} \right)^{1/2}$$

Neutron Stars and Black Holes

- Both the maximal rotation frequency and the typical pulsation frequency of white dwarfs fall short so we are left with neutron stars and black holes.
- Isolated black holes have no structure to emit periodically and material in orbit around a BH would spiral in and the period would decrease.
- Ditto for neutron star binaries
- Pulsation modes of a neutron star fit the bill for the period, BUT the period would typically decrease as the energy in the mode dissipates.

The Big Flywheel

- If a neutron star is born spinning near break-up, it has as much rotational energy as a supernova.
- If there only was a way to convert that energy into radio waves.
- Hmmmm……

Magnetic Dipole Radiation (1)

- Regardless of what's going on inside of the star, the magnetic dipole moment is

$$|\mathbf{m}| = \frac{\mathbf{B_p R^3}}{\mathbf{2}}$$

where B_p is the strength of the dipole field at the pole.

- If the dipole moment varies with time, energy is radiated at a rate of

$$\dot{E} = -\frac{2}{3c^3}|\ddot{\mathbf{m}}|^2$$

- Suppose that the magnetic axis is not aligned with the rotation axis (α is the angle between the axes).

$$\mathbf{m} = |\mathbf{m}| \begin{bmatrix} \cos\alpha \\ \sin\alpha\cos\phi \\ \sin\alpha\sin\phi \end{bmatrix}$$

$$\phi = \Omega t$$

$$|\ddot{m}| = \Omega^2 \sin\alpha |\mathbf{m}|$$

$$\dot{E} = -\frac{B_p^2 R^6 \Omega^4 \sin^2\alpha}{6c^3}$$

Magnetic Dipole Radiation (2)

- The total rotational energy of the star is

$$E = \frac{1}{2} I \Omega^2, \dot{E} = I \Omega \dot{\Omega}$$

- Putting things together

$$\dot{\Omega} = -\frac{B_p^2 R^6 \Omega^3 \sin^2 \alpha}{6 c^3 I}$$

- Let's define a characteristic time,

$$T = -\frac{\Omega_0}{\dot{\Omega}_0} = \frac{6 c^3 I}{B_p^2 R^6 \Omega_0^2 \sin^2 \alpha}$$

- This gives us

$$\dot{\Omega} = -\frac{\Omega}{T} \left(\frac{\Omega}{\Omega_0} \right)^2$$

- Separating and integrating,

$$\frac{1}{\Omega^3} d\Omega = -\frac{1}{T \Omega_0^2} dt$$

$$\frac{1}{2\Omega_0^2} - \frac{1}{2\Omega_i^2} = \frac{t_0 - t_i}{T \Omega_0^2}$$

- Let's assume that at t_i, P=0,

$$\frac{1}{2\Omega_0^2} = \frac{t_0 - t_i}{T \Omega_0^2}; \quad \tau = t_0 - t_i = \frac{1}{2} T$$

P and P-dot

- Although theoretically it is natural to talk about the frequency, observationally people talk about the period, $P=2\pi/\Omega$ and $dP/dt=-2\pi/\Omega^2\, d\Omega/dt$, a.k.a. P-dot.

$$T = -\frac{\Omega_0}{\dot{\Omega}_0} = \frac{P}{\dot{P}}$$

- If you can estimate I and R, you can get an estimate of B_p

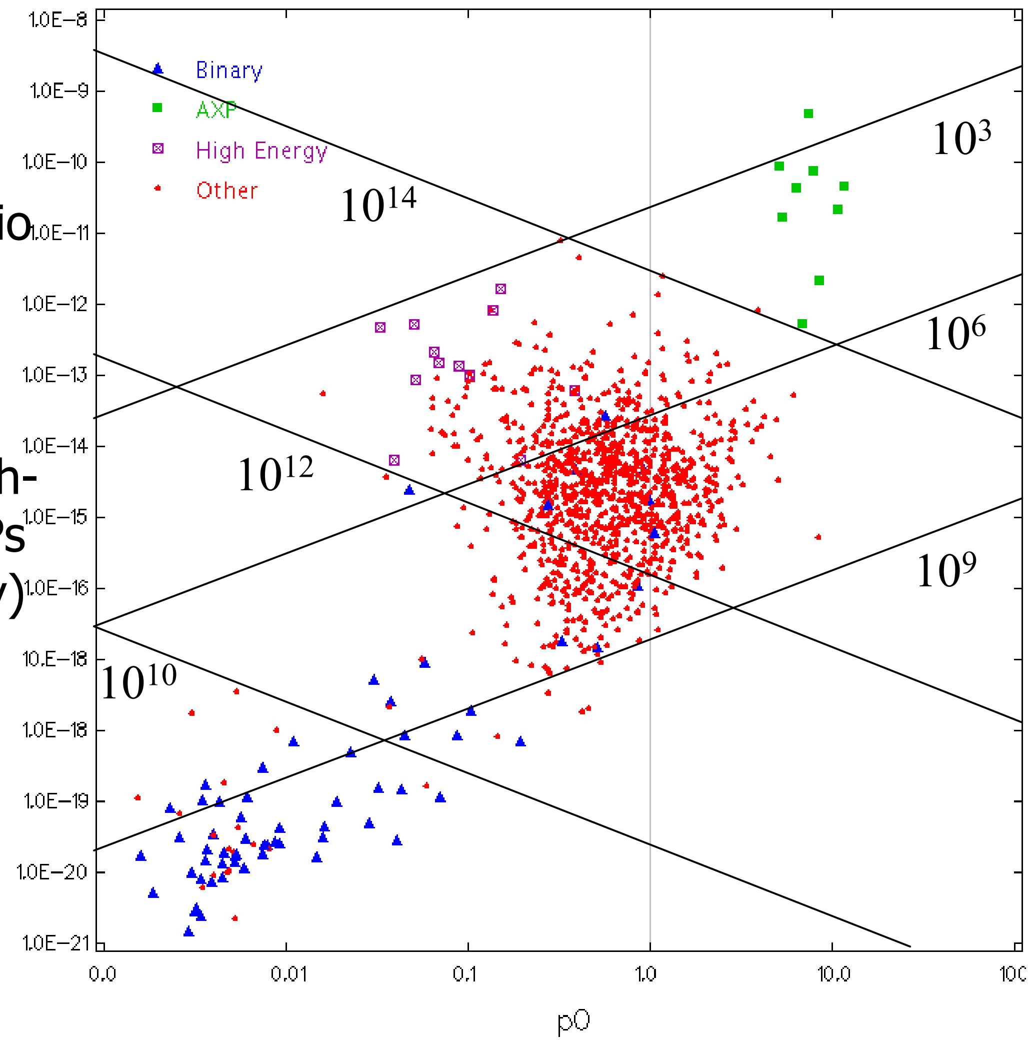
$$B_p^2 \sin^2\alpha = \frac{6c^3 I P\dot{P}}{4\pi^2 R^6}$$

$$B_p \sin\alpha = 6.4\times 10^{19} I_{45} R_6^{-6} (P_1 \dot{P})^{1/2}\ \mathrm{G}$$

- Some examples:
- Crab: P=0.033s, P-dot=4 x 10^{-13}
  - B_p= 7 x 10^{12} G, $T/2$=1300 yr
- Vela: P=0.089s, P-dot=1 x 10^{-13}
  - B_p= 6 x 10^{12} G, $T/2$=14000 yr
- 1841: P=11.77s, P-dot=4 x 10^{-11}
  - B_p= 1 x 10^{15} G, $T/2$= 4700 yr
- 1937: P=0.0016s, P-dot=1 x 10^{-19}
  - B_p= 8 x 10^{8} G, $T/2$=2.5 x 10^{8} yr

The P-P-dot Diagram!

- Like the H-R diagram.
- Things to notice:
  - most red dots (isolated radio pulsars): 10^{11-13} G, 10^{5-8} yr
  - most PSRs in binaries have short periods
  - Many young PSRs have high-energy emission or are AXPs (no radio and thermal x-ray)



Another Model (GW)

- A spinning barbell emits gravitational radiation and slows according to

$$\dot{E} = -\frac{32G}{5\ c^5} I^2 \epsilon^2 \Omega^6$$

- Astronomers like power-law models, so take

$$\dot{\Omega} = -A\Omega^n$$

- How can we determine *n*?
  - *n*=3: MD, *n*=5: GW

- Take the time derivative of both sides,

$$\ddot{\Omega} = -An\Omega^{n-1}\dot{\Omega}$$

$$\Omega\ddot{\Omega} = -nA\Omega^n\dot{\Omega} = n\dot{\Omega}^2$$

$$n = \frac{\Omega\ddot{\Omega}}{\dot{\Omega}^2} \quad \tau = \frac{T}{n-1}$$

- Unfortunately, *n* is difficult to measure accurately but there is other evidence for the MD model.

Evidence for Dipole Model

- Measurement of magnetic field strengths from cyclotron lines on Her X-1 gives 4×10^{12} G.
- Energy from spin-down of Crab is sufficient to power the Crab nebula.
- Polarization of the radiation is characteristic for a magnetic dipole geometry.

Pulsar Emission Observed

- The individual pulses are quite random.
- The sum of many pulses is constant for a particular pulsar.
- The emitting elements are all in a particular region but not all are active at the same time.