E & J #1

ORGANIZING CONCEPTS

$\boxed{\text{ENERGY AND ANGULAR MOMENTUM}}$

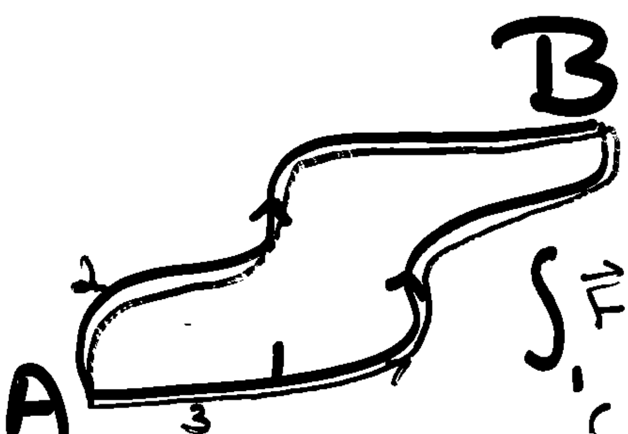
- EXPLAIN UNDER WHAT CIRCUMSTANCES ARE ENERGY, MOMENTUM AND ANGULAR MOMENTUM CONSERVED.
- DETERMINE THE EQUATIONS OF MOTION WITH LAGRANGE'S EQUATIONS

Ed J

#2

CONSERVATIVE FORCES

- In one dimension, all forces that depend only on position are conservative.
- In one dimension, there is only one way from A → B
- For a force to be conservative, the work must not depend on path.



$$\int_1 \vec{F}\cdot d\vec{s} - \int_2 \vec{F}\cdot d\vec{s} = 0$$

$$\oint_3 \vec{F}\cdot d\vec{s} = 0$$

Stokes' theorem

$$\int (\vec{\nabla}\times\vec{F})\cdot d\vec{A} = 0$$

- For this to be true for all paths $\boxed{\vec{\nabla}\times\vec{F} = 0}$

E & $\vec{J}$ #3

TORQUES & ANGULAR MOMENTUM

- Torque is defined as

$$\vec{\tau} = \vec{r} \times \vec{F}$$

- Angular momentum is

$$\vec{J} = \vec{r} \times \vec{p}$$

$$\frac{d\vec{J}}{dt} = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} = \vec{r} \times \vec{F} = \vec{\tau}$$

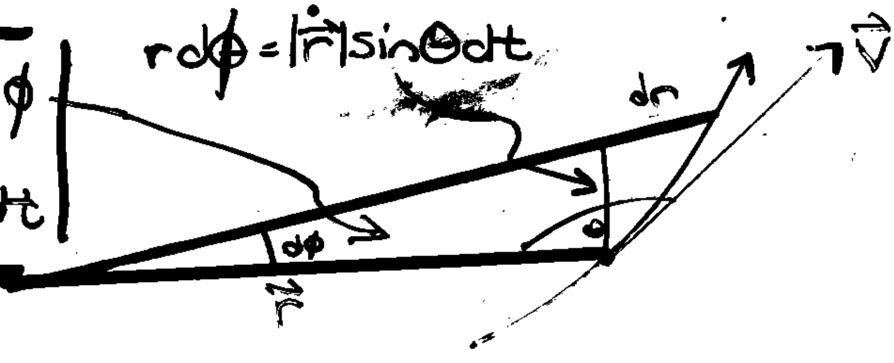
- Both the direction and magnitude are constant.

$\vec{r}, \vec{p}$ define a plane. This plane is constant.

$$|\vec{J}| = m r |\dot{\vec{r}}| \sin\theta = m r v_\phi = m r^2 \dot{\phi}$$

$$\boxed{\begin{aligned} dA &= \frac{1}{2} r^2 \, d\phi \\ dA &= \frac{1}{2} \frac{|\vec{L}|}{m} \, dt \end{aligned}}$$

$r\,d\phi = |\dot{\vec{r}}| \sin\theta \, dt$



E&J #4

CALCULUS OF VARIATIONS

- What is the shape of the fastest roller coaster?
- What is the shape of a soap bubble between to rings?

And believe it or not

- What is the trajectory of a ball through the air?

What do these questions have in common?

E&J #5

They require us to find a function that minimizes an integral.

Calculus helps us to find the minimum value of a particular function

Answering the first question spurred Newton to create the CALCULUS OF VARIATIONS

E&G #7

THE AMAZING THINGS

- Particles and fields evolve in time so as to minimize (extremize)

$$S = \int (T-V)\, dt = \int L\, dt$$

just by following $\ddot{x} = -\frac{V'}{m}$

- An analogy to Fermat's principle tells you that particles accumulate a phase

$$\boxed{\begin{array}{l} \Delta\phi \propto \int_{t_1}^{t_2} (T-V)\, dt \\ \Delta\phi = \int dt \times \omega \end{array}} \quad \Delta\phi = \frac{S}{\hbar}$$

EL3 #6

THE MAIN RESULT

The function $y(x)$ that minimizes

$$I = \int_{x_0}^{x_1} f(y, y')\,dx$$

and satisfies $y(x_0) = y_0$, $y(x_1) = y_1$ also satisfies the differential equation

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$$

EULER-LAGRANGE EQUATION

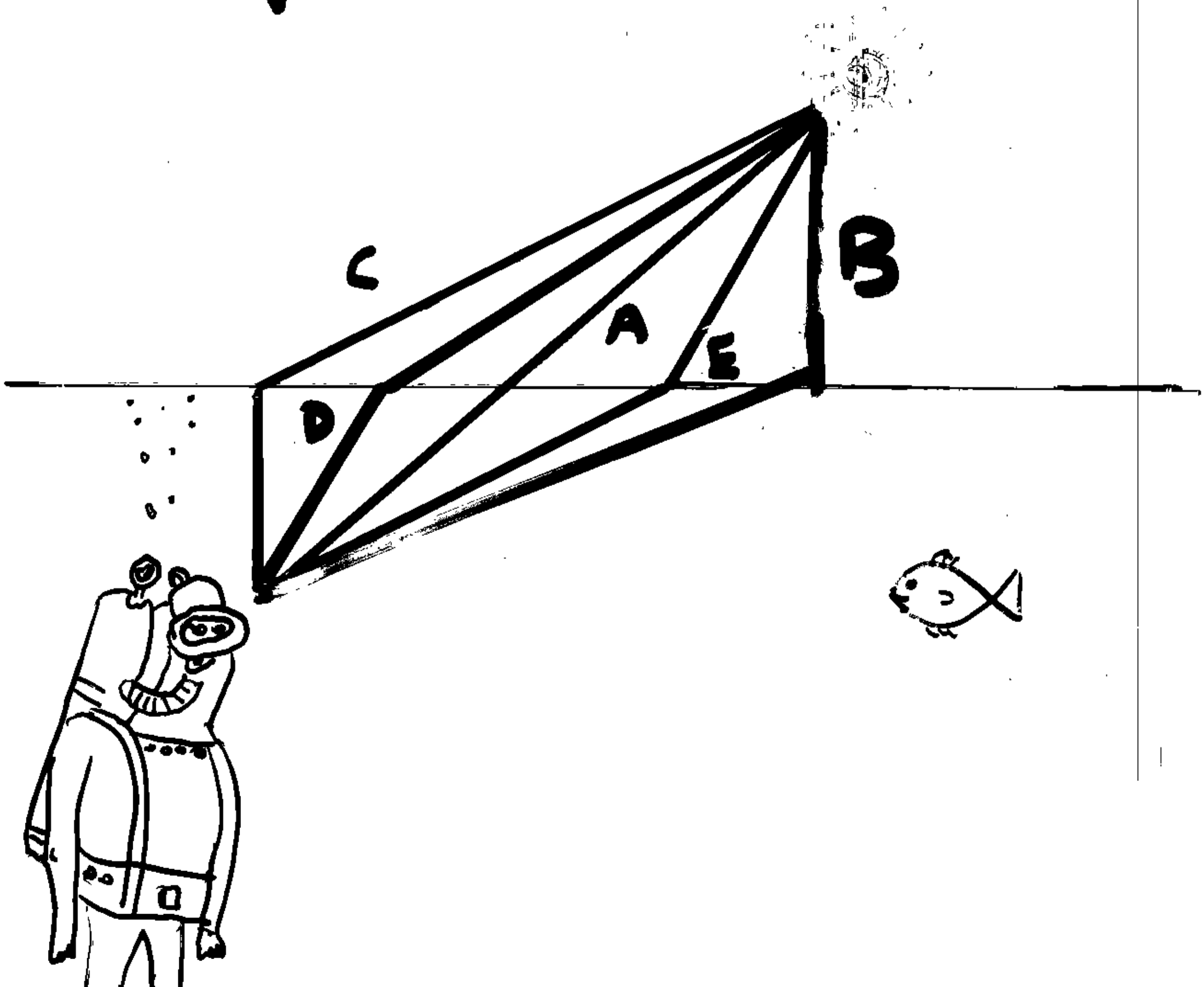
and vice versa.

Integral Statement $\Leftrightarrow$ Differential Equation

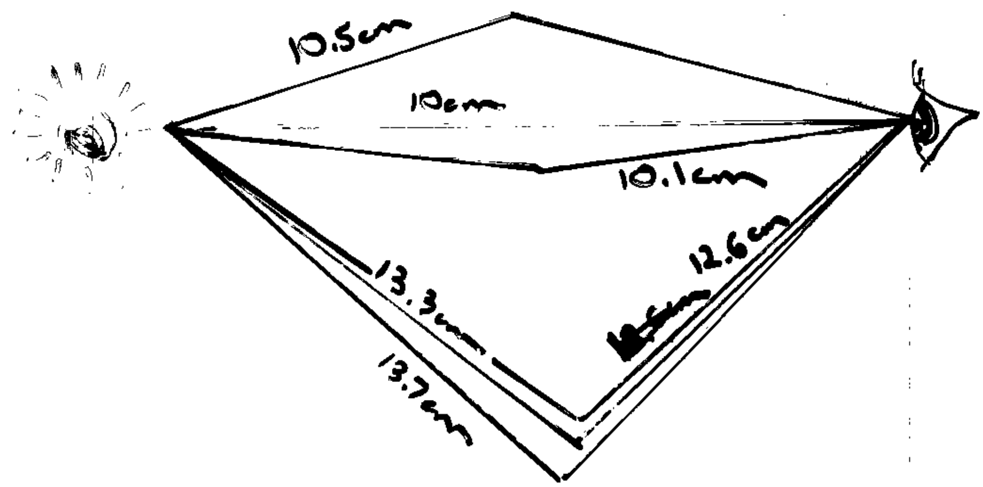
E&3 #8

WHAT IS FERMAT'S PRINCIPLE?

- Light takes the path that minimizes the time between two points.



WHY DOES LIGHT APPEAR TO GO IN STRAIGHT LINES WHEN WAVES GO EVERYWHERE?



- The three paths near the straight one diverge by quite a bit but their lengths are nearly the same (constructive interference)
- Although the other paths are really close, their lengths differ by a lot.

E&J #10

HOW TO SEE WAVES?

- When two different paths differ in length by about a wavelengths, you get interference.

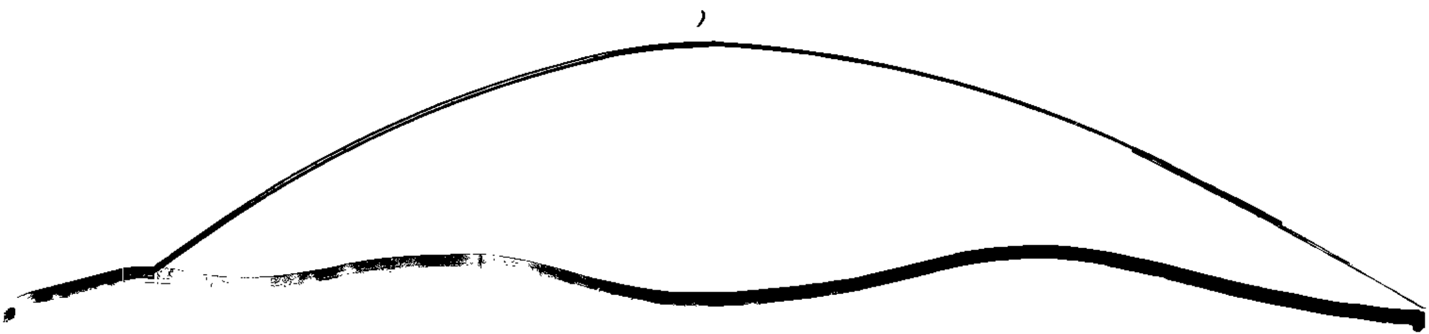
HOW DOES THIS TRANSLATE TO MECHANICS?

- If the difference in the action along two paths differs by less than some number, you get interference.

WHAT IS THE QUANTUM OF ACTION?

HOW COULD YOU MEASURE THIS?

BACK TO BASEBALL!



- This is the path of a baseball through the air. Imagine that you shrunk the baseball down to the mass of an atom.

$$S = m\int\left(\frac{v^2}{2} - gy\right)dt$$

EGJ

ANOTHER ROUTE TO QUANTUM MECHANICS

- Take all possible paths and weigh them according to

$$e^{iS/\hbar}$$

Add the contributions of the paths as complex numbers to find what we can observe.

- Discovered by Richard Feynman
- Basis of quantum field theory.