MATH 312: ASSIGNMENT 1
DUE DATE: OCTOBER 12, 2012

1) Use the division algorithm (write all steps clearly) to find the greatest common divisor of (20785,44350).

2) Use the extended Euclidean algorithm to express the greatest common divisor of (34709, 100313) as a linear combination of the two integers.

3) Show that if k is a positive integer, then $(3k + 2)$ and $(5k + 3)$ are relatively prime.

4) Suppose that two players begin with a pair of positive integers and take turns making moves of the following type. A player can move from the pair or positive integers $\{x, y\}$ with $x \geq y$ to any of the pairs $\{x - ty, y\}$ where t is a positive integer and $x - ty \geq 0$. A winning move consists of moving to a pair with one of the entries being equal to 0. Show that every sequence of moves $\{a, b\}$ must eventually end with the pair $\{0, (a, b)\}$.

5) Let $S = \{a + b\sqrt{-5} \,|\, a,\ b \in \mathbb{Z}\}$. If $\alpha = a + b\sqrt{-5}$, let $N(\alpha) = \alpha\bar{\alpha}$ where $\bar{\alpha}$ is the conjugate $(a - b\sqrt{-5})$. Show that if $\alpha,\ \beta$ are in S then $N(\alpha\beta) = N(\alpha)N(\beta)$.

6) An element α in the set S above is prime if it cannot be written as a product of two numbers in S without one of them being equal to ± 1. Show that 2 is a prime number in S and factor 21 into a product of primes in S. Can you factor it in more than one way?

7) Find the least common multiple and greatest common divisor of 343 and 999 using the fundamental theorem of arithmetic, in detail.

Show that if a and b are positive integers with $(a, b) = 1$ then $(a^n, b^n) = 1$ for all positive integers n.

8) Show that $\sqrt{2} + \sqrt{3}$ is irrational.

9) Show that $\log_2 3$ is irrational.

10) Suppose a and b are two positive integers. When is $(a, b) = [a, b]$?

Date: October 5, 2012.