

1. By finding the curl, determine which of the following forces are conservative:

(a)  $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

(b)  $\mathbf{F} = y\mathbf{i} - x\mathbf{j} + z^2\mathbf{k}$

(c)  $\mathbf{F} = y\mathbf{i} + x\mathbf{j} + z^3\mathbf{k}$

(d)  $\mathbf{F} = -kr^{-n}\mathbf{e}_r$  in spherical coordinates

2. Particles of mud are thrown from the rim of a rolling bicycle wheel. If the forward speed of the bicycle is  $v_0$ , and the radius of the wheel is  $b$ , show that the greatest height above the ground that the mud can go is

$$b + \frac{v_0^2}{2g} + \frac{gb^2}{2v_0^2}.$$

At what point on the rolling wheel does this mud leave? (Note: it is necessary to assume that  $v_0^2 \geq bg$ .)

3. A cannon that is capable of firing a shell at speed  $v_0$  is mounted on a vertical tower of height  $h$  that overlooks a level plain below. Ignoring air resistance, show that the elevation angle  $\alpha$  at which the cannon must be set to achieve maximum range is given by the expression

$$\csc^2 \alpha = 2 \left( 1 + \frac{gh}{v_0^2} \right).$$