Name:

C. Brun,

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Mathematics 100-180

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Marks

Short-Answer Questions. Questions 1 and 2 are short-answer questions. Put your answer in the box provided. Full marks will be given for a correct answer placed in the box. Show your work also, for part marks. Each part is worth 3 marks, but not all parts are of equal difficulty. Simplify your answers as much as possible in Questions 1 and 2.

[9] 1. Determine whether each of the following limits exists, and find the value if they do. If a limit below does not exist, determine whether it "equals" ∞ , $-\infty$, or neither.

(a) [3]
$$\lim_{x \to -1} \frac{x^2 - x - 2}{x^2 - 1}$$

o lim	(x+1)(x-	2)
7->-1	(+FT)(1-	

Answer

(b)	$[3] \lim_{t \to 1} \frac{}{}$	$\frac{t^2+8-3}{t-1}$	JE381
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Answer 2 3

(c)	[3]	lim ~→0~	$\left(\frac{1}{x}\right)$	*****	$\frac{1}{ x }$
			$\frac{1}{\lambda}$		$\frac{1}{\lambda}$

Answer

- [12] **2.**
- (a) [3] Find $\frac{d}{dx} \left(\frac{x^4 + x^{7/2}}{x^2} \right)$. Remember (see the instructions above Question 1) that your answers must be completely simplified in Questions 1 and 2.



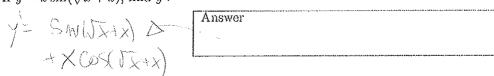
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(b) [3] If $y = x^2 \cos x$, find the second derivative y''. Express your answer in the form $p(x) \sin x + q(x) \cos x$ where p(x) and q(x) are polynomials.

$$y' = 2 \times \omega_{5x} + x^{2} (-s_{1Nx}) \qquad \frac{Answer}{-4 \times (-s_{1Nx})} + (2x^{2}) \cos_{x}$$

$$y'' = 2 \cos_{x} + 2x(-s_{1Nx}) + \omega_{x} - 2x + s_{1Nx} + x^{2} \cos_{x}$$

(c) [3] If $y = x \sin(\sqrt{x} + x)$, find y'.



(d) [3] f is a function that satisfies f'(e) = e. Let $g(x) = f(e^{x^2})$. Find g'(1).

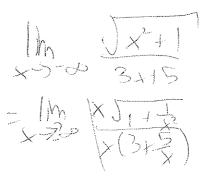
$$g'(x) = 2xe^{x^2} f'(e^{x^2})$$
Answer
$$2e^2$$

Full-Solution Problems. In questions 3-7, justify your answers and show all your work. If a box is provided, write your final answer there. Unless otherwise indicated, simplification of answers is not required in these questions.

- [7] 3. Let $f(x) = \frac{\sqrt{x^2 + 1}}{3x + 5}$.
 - (a) [4] Determine the horizontal asymptotes of the graph y = f(x).

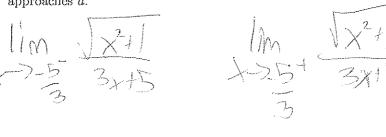
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(b) [3] Determine the vertical asymptote(s) of the graph y = f(x). For each vertical asymptote x = a, determine whether each of the one-sided limits "equals" ∞ or $-\infty$ as x approaches a.





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[6] Let a and b be constants, and define

$$f(x) = \begin{cases} (x^2 + b) & \text{if } x < 1\\ ax + b & \text{if } 1 \le x \le 2\\ 5x - 3 & \text{if } x > 2 \end{cases}$$

Find the values of a and b for which f is continuous at x = 1 and x = 2. Fully justify your answer.

	 ····	
ınswer		

|+b=a+b| 2a+b=10-3 |-a| 2-1b=7 |-b|

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[4] 5. Prove that the equation $x^3 - 3x + 1 = 0$ has at least two positive real solutions. Carefully cite any theorem you use, and justify why the theorem can be used.

$$f(0) = 1$$

 $f(1) = -1$
 $f(2) = 3$
 $f(-3) = -27+9+1$

[6] 6. Find the point(s) on the curve $y = x^2 + 6$ such that the tangent line(s) to the curve at these point(s) pass through the point (2,1).

Y=2x Y=4.

Y=mx+b 1=4.2+b -7=b Answer Y = 4 × 7

Use the definition of the derivative in the questions below. No marks will be given for any [6] other method. In this and any other question, you may use the back of a page if necessary, but please indicate so.

(a) [3] Find

$$\frac{d}{dx}\left(\frac{1}{1-x}\right)$$

(b) [3] Determine whether the function

$$f(x) = \begin{cases} x^3 \sin\frac{1}{x} & \text{if } x < 0\\ 4x^2 - x & \text{if } x \ge 0 \end{cases}$$

is differentiable at x = 0. $(3x^{2}SNx) + (-x^{-2})COB(x)x^{3} = 8x - 1/6$ $3x^{2}SN(x) - x Cos(x)/6 = 8x - 1/6$

from left, as try tradions are build mabsolute values by 1, this is 0. As 0 7-1, this isnot differentiable