5 Practice Questions and Useful Tips

1) Divide $x^2 - 9x - 10$ by $x + 1$

$$\begin{array}{rl} x+1 \big) & \overline{x^2 \quad -9x-10} \end{array}$$

Divide the leading x^2 inside by the leading x in front. You get an x. So put x on top.

$$\begin{array}{rl} & \quad x \\ x+1 \big) & \overline{x^2 \quad -9x-10} \end{array}$$

Now take that x, and multiply it through the divisor, $x + 1$. First, multiply the x (on top) by the x (on the "side"), and carry the x^2 underneath:

$$\begin{array}{rl} & \quad x \\ x+1 \big) & \overline{x^2 \quad -9x-10} \\ & x^2 \end{array}$$

Then multiply the x (on top) by the 1 (on the "side"), and carry the $1x$ underneath:

$$\begin{array}{rl} & \quad x \\ x+1 \big) & \overline{x^2 \quad -9x-10} \\ & x^2 \quad +1x \end{array}$$

To subtract the polynomials, change all the signs in the second line

$$\begin{array}{rl} & \quad x \\ x+1 \big) & \overline{x^2 \quad -9x-10} \\ & \underline{-x^2 \quad -1x \qquad\quad} \end{array}$$

Then add down. The first term (the x^2) will cancel out:

$$\begin{array}{rl} & \quad x \\ x+1 \big) & \overline{x^2 \quad -9x-10} \\ & \underline{-x^2 \quad -1x \qquad\quad} \\ & \qquad -10x \end{array}$$

Carry down that last term, the "subtract ten", from the dividend:

$$\begin{array}{rl} & \quad x \\ x+1 \big) & \overline{x^2 \quad -9x-10} \\ & \underline{-x^2 \quad -1x \qquad\quad} \\ & \qquad -10x-10 \end{array}$$

Look at the *x* from the divisor and the new leading term, the –10*x*, in the bottom line of the division. Divide the –10*x* by the *x*, and you end up with a –10, so put that on top:

$$\begin{array}{r|l} & x - 10 \\ \hline x+1 & x^2 - 9x - 10 \\ & -x^2 - 1x \\ \hline & \quad -10x - 10 \end{array}$$

Now multiply the –10 (on top) by the leading *x*(on the "side"), and carry the –10*x* to the bottom:

$$\begin{array}{r|l} & x - 10 \\ \hline x+1 & x^2 - 9x - 10 \\ & -x^2 - 1x \\ \hline & \quad -10x - 10 \\ & \quad -10x \end{array}$$

Multiply the –10 (on top) by the 1 (on the "side"), and carry the –10 to the bottom:

$$\begin{array}{r|l} & x - 10 \\ \hline x+1 & x^2 - 9x - 10 \\ & -x^2 - 1x \\ \hline & \quad -10x - 10 \\ & \quad -10x - 10 \end{array}$$

Draw the equals bar, and change the signs on all the terms in the bottom row

$$\begin{array}{r|l} & x - 10 \\ \hline x+1 & x^2 - 9x - 10 \\ & -x^2 - 1x \\ \hline & \quad -10x - 10 \\ & \quad +10x + 10 \\ \hline \end{array}$$

Add down:

$$\begin{array}{r|l} & x - 10 \\ \hline x+1 & x^2 - 9x - 10 \\ & -x^2 - 1x \\ \hline & \quad -10x - 10 \\ & \quad +10x + 10 \\ \hline & \qquad\qquad 0 \end{array}$$

Then the solution to this division is: ***x* – 10**

Since the remainder on this division was zero (that is, since there wasn't anything left over), the division came out "even". In the case of the above polynomial division, the zero remainder tells us that $x + 1$ is a factor of $x^2 - 9x - 10$, which you can confirm by factoring the original quadratic dividend, $x^2 - 9x - 10$.

2) **Simplify** $\dfrac{x^2 + 9x + 14}{x + 7}$

Factor and then cancel the common factor:

$$\frac{x^2 + 9x + 14}{x + 7} = \frac{(x+2)(x+7)}{x+7} = x + 2$$

Or use long division:

$$\begin{array}{r l}
 & x + 2 \\
x+7 \big) & x^2 + 9x + 14 \\
 & -x^2 \mathbin{\overset{-}{+}} 7x \\
\hline
 & 2x + 14 \\
 & -2x \mathbin{\overset{-}{+}} 14 \\
\hline
 & 0
\end{array}$$

Don’t forget to change signs as you subtract.

The answer: ***x* + 2**

3) **Divide $3x^3 - 5x^2 + 10x - 3$ by $3x + 1$**

$$\begin{array}{r l}
 & x^2 \quad - 2x \quad + 4 \\
3x+1 \big) & 3x^3 - 5x^2 + 10x - 3 \\
 & -3x^3 \mathbin{\overset{-}{+}} 1x^2 \\
\hline
 & - 6x^2 + 10x - 3 \\
 & + 6x^2 + 2x \\
\hline
 & 12x - 3 \\
 & -12x \mathbin{\overset{-}{+}} 4 \\
\hline
 & -7
\end{array}$$

This division did not come out even. What do you do with the remainder?

Since the remainder is –7 and since the divisor is $3x + 1$, then turn the remainder into a fraction (the remainder divided by the original divisor), and add this fraction to the polynomial across the top of the division symbol. Since the division looks like this:

$$
\begin{array}{r}
x^2 \quad -2x \quad +4 \\
3x+1\overline{)\,3x^3 - 5x^2 + 10x - 3} \\
\underline{\color{red}{-}3x^3 \mp 1x^2 \qquad\qquad\quad} \\
-6x^2 + 10x - 3 \\
\underline{\color{red}{+}6x^2 \color{red}{+} \;\; 2x \qquad} \\
12x - 3 \\
\underline{\color{red}{-}12x \mp 4} \\
-7
\end{array}
$$

...then the answer is:

$$x^2 - 2x + 4 + \frac{-7}{3x+1}$$

Do not write the polynomial "mixed number" in the same format as numerical mixed numbers! If you just append the fractional part to the polynomial part, this will be interpreted as polynomial multiplication, which is not what you mean!

4) **Divide $2x^3 - 9x^2 + 15$ by $2x - 5$**

First off, note that there is a gap in the degrees of the terms of the dividend: the polynomial
$2x^3 - 9x^2 + 15$ has no x term. It is important to leave space for a x-term column, just in case. You can create this space by turning the dividend into $2x^3 - 9x^2 \color{red}{+ 0x} + 15$. This is a legitimate mathematical step: since I've only added zero, I haven't actually changed the value of anything. Do the division:

$$
\begin{array}{r}
x^2 \quad -2x \quad -5 \\
2x-5\overline{)\,2x^3 - 9x^2 + 0x + 15} \\
\underline{\color{red}{-}2x^3 \color{red}{+} 5x^2 \qquad\qquad\quad} \\
-4x^2 + 0x + 15 \\
\underline{\color{red}{+}4x^2 \mp 10x \qquad} \\
-10x + 15 \\
\underline{\color{red}{+}10x \mp 25} \\
-10
\end{array}
$$

Remember to *add* the remainder to the polynomial part of the answer:

$$x^2 - 2x - 5 + \frac{-10}{2x-5}$$

5) **Divide $4x^4 + 3x^3 + 2x + 1$ by $x^2 + x + 2$**

Add a $0x^2$ term to the dividend (inside the division symbol):

$$
\begin{array}{rl}
 & 4x^2 \quad - x \quad - 7 \\
x^2+x+2 & \overline{)\,4x^4 + 3x^3 + 0x^2 + 2x + 1} \\
 & -4x^4 - 4x^3 - 8x^2 \\
\hline
 & \qquad -x^3 - 8x^2 + 2x + 1 \\
 & \qquad +x^3 + x^2 + 2x \\
\hline
 & \qquad\qquad -7x^2 + 4x + 1 \\
 & \qquad\qquad +7x^2 + 7x + 14 \\
\hline
 & \qquad\qquad\qquad 11x + 15
\end{array}
$$

The answer is:

$$4x^2 - x - 7 + \frac{11x+15}{x^2+x+2}$$