

## ASSIGNMENT 5

DUE DATE: NOV 8, 2011

- 1) If  $N$  is a normal subgroup of  $G$ ,  $P$  is a  $p$ -Sylow subgroup of  $N$ , then  $PN/N$  is a  $p$ -Sylow subgroup of  $G/N$ . 5 points
- 2) Prove that a group of order 28 with a normal subgroup of order 4 is abelian. 5 points
- 3) An action of a group  $G$  on  $X$  is said to be transitive if given any two elements  $x, y$  in  $X$ , there is an element  $g \in G$  such that  $gx = y$ . Give two examples of transitive group actions with full details. 5 points
- 4) Show that every nonabelian group of order  $2p$  where  $p$  is a prime is dihedral. 5 points  
(Hint: Use Sylow theorems and the fact that in  $\mathbb{Z}/p$ , if  $x^2 = 1$ , then  $x = \pm 1$ .)
- 5) Let  $G$  be a noncyclic group of order 21. How many 3-Sylow subgroups does  $G$  have?  
Prove that  $G$  has 14 elements of order 3. 5 points