ASSIGNMENT 2

DUE: OCTOBER 4, 2011

1 (10 points) Prove that if G is abelian group, then for all $a, b \in G$ we have $(a \cdot b)^n = a^n \cdot b^n$.

2 (10 points) Show that if every element in a group G is its own inverse, then G is abelian.

3 (10 points) If $a \in G$, then define $N(a) = \{x \in G \mid xax^{-1} = a.\}$. Show that $N(a)$ is a subgroup of G. (It is called the normaliser or centraliser subgroup in G of a).

4 (10 points) The center of a group G is the set of all elements $z \in G$ such that $zg = gz$ for all $g \in G$. Show that the center is an abelian subgroup of G.

5 (10 points) If H is a subgroup of G, then the normaliser of H denoted $N(H)$ is the set of all elements g in G such that $gHg^{-1} = H$. The centraliser of H denoted $C(H)$ is the set of all elements g in G such that $gh = hg$ for all elements h in H.

(a) Show that $N(H)$ and $C(H)$ are subgroups of G.

(b) What, if any, is the containment relation between these two subgroups?