

# UBC MATH 312 QUIZ 1

1/10/12

## QUESTION 3

I'm glad to see that most of you have a good "feel" for proofs by mathematical induction.

No one really had a problem with the Base Case for this question:  $4^2 = 16 < 24 = 4!$

But most of you scored 3 out of 4 because your Inductive Step was not thorough enough, although I did give 4 out of 4 for some of the more convincing attempts.

Here is an example of a more thorough proof from one of your classmates. You may find it useful as reference. I've only re-produced the Inductive Step portion here:

### INDUCTIVE STEP

Assume that  $n^2 < n!$  for some  $n > 4$ ,  $n \in \mathbb{N}$ . Then,

$$\begin{aligned} & (n+1)^2 \\ &= n^2 + 2n + 1 \\ &< n! + 2n + 1 \end{aligned}$$

{ by assumption }

$$< n! + (n-1) \cdot n! + n!$$

{ since for  $n \geq 4$ , we have

$$= n!(1 + n - 1 + 1)$$

$$(n-1) > 2$$

$$= n!(n+1)$$

$$n! > n$$

$$= (n+1)!$$

$$\text{and } n! > 1 \}$$

□