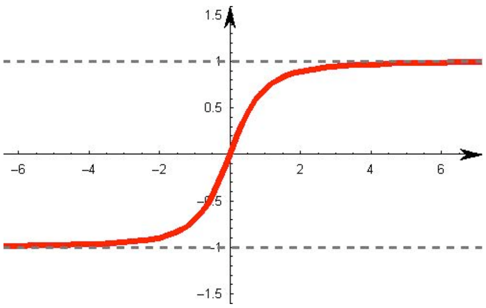
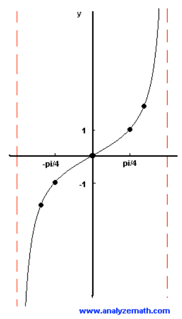
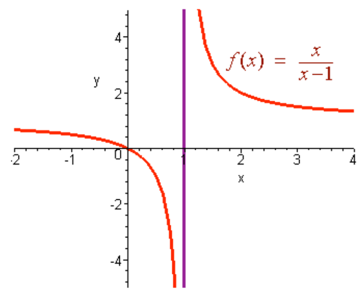
Polynomial Long Division and Finding Vertical, Horizontal and Slant Asymptotes

An **asymptote** is a line that a function approaches but never touches as it stretches to positive or negative infinity.







Vertical asymptotes stand parallel to the y- axis. They are obtained by determining the value(s) of x that make the function undefined. **When working with rational functions, the denominator is equated to 0 and solved for x to find vertical asymptotes.**

Ex 1.

Find the vertical asymptotes:

$$y = \frac{4x+1}{3x+3}$$

The first thing we need to understand when given a fraction is that the **denominator can never be 0**. Dividing by 0 is undefined. This means that any value of x that causes the devisor to equal 0 is an asymptote. The function cannot have the said value of x and therefore does not pass any points with that particular value; the function is undefined at this point. All of the points with this value of x line up to form a perfectly vertical line which can never be part of a function, otherwise known as a vertical asymptote.

To find the vertical asymptote, the denominator must be equated to zero and solved for x.

$3x+3=0$
$x=-3/3$
$x= -1$

The function has a vertical asymptote at **x = -1**

Ex 2.

$$y= \frac{(7x - 24)^2}{2x^3 + 3x^2 - 8x - 12}$$

To find the values of x that produces a value of 0 in the denominator, we must first factor since when either of the factors of any function equals zero, the entire function equals zero. Factoring, we get:

$$y= \frac{(7x-24)^2}{(2x+3)(x^2-4)}$$

- ⇨ $(2x+3)(x^2-4) = 0$
- ⇨ $2x+3=0$
- ⇨ $2x = -3$
- ⇨ $x = -3/2$

$x^2 - 4 = 0$
$x^2 = 4$
$x = \pm 2$

Therefore, the function has vertical asymptotes at x = - 3/2, - 2, 2.

SAMPLE QUESTIONS:

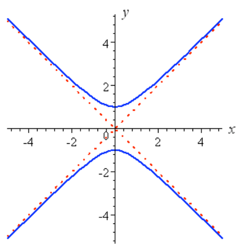
Find the equation of the vertical asymptotes of the following functions:

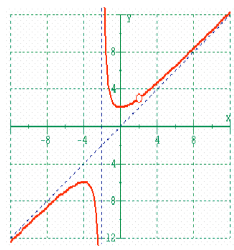
1. $f(x) = \frac{10x^2 + 3}{(3x-4)(2-x)}$

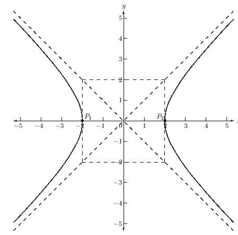
2. $g(x) = \frac{4x}{x^2 - 8x - 20}$

3. $f(x) = \frac{8}{x(x^2 + 6x + 9)}$

4. $g(x) = \frac{6x^2}{(x+6)(x^2-9x+20)}$







Slant asymptotes are lines that are not parallel to the x- or y- axis that a function may approach but will never reach or pass. Slant asymptotes are observed in rational functions where the degree of the leading polynomial in the numerator is one higher than the degree of the polynomial in the denominator. When these polynomials are divided, the quotient will represent a slant asymptote to the function.

[Ex. 3]

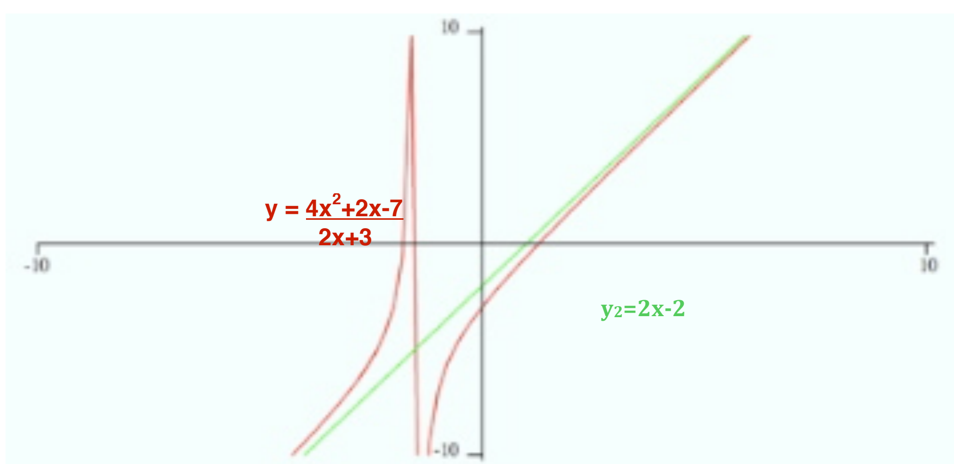
Find the equation for the slant asymptote for the following functions:

a) $y = \dfrac{4x^2+2x-7}{2x+3}$

SOLUTION:

(perform polynomial long division)

$$\begin{array}{r} 2x-2 \\ 2x+3 \;/\; 4x^2+2x-7 \\ \underline{4x^2-6x} \\ -4x-7 \\ \underline{-4x-6} \\ -1 \end{array}$$



After doing polynomial long division, we see that the quotient equals:

$$\frac{4x^2+2x-7}{2x+3} = 2x - 2 - \frac{1}{4x^2+2x-7}$$

The remainder portion of the quotient is arbitrary in the equation of the slant asymptote since as x approaches $\pm$ infinity, the value of the remainder approaches zero.

Therefore, **y= 2x-2** represents the slant asymptote for this graph.

b) $y = \dfrac{5x^2-2x+5}{x+1}$

(*perform polynomial long division*)

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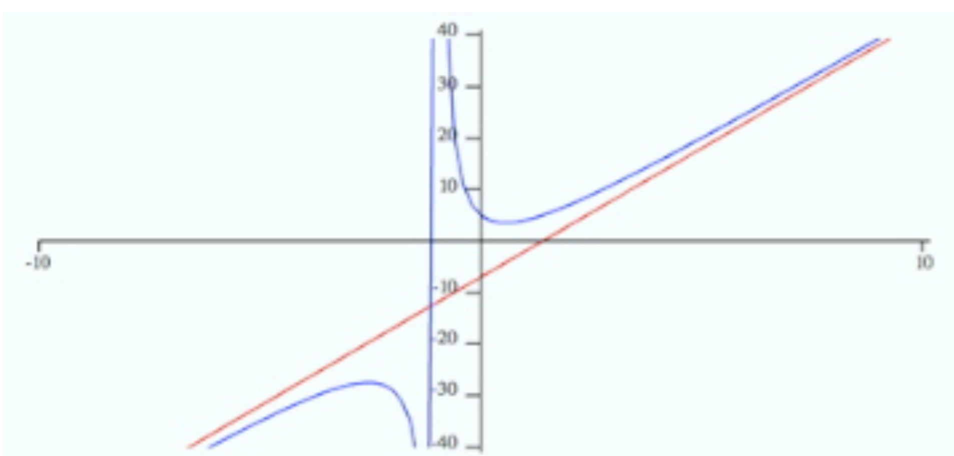
          5x - 7
x+1  / 5x² - 2x + 5
       5x² + 5x
            -7x + 5
            -7x – 7
                 12
```

The equation for the slant asymptote is y = 5x-7

Graphing the asymptote (red) and the original function (blue), we see:

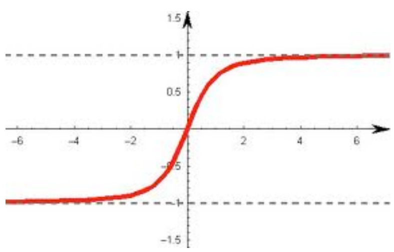


The function seems to come very close to the asymptote but never touches this line as it extends to positive and negative infinity.

SAMPLE QUESTIONS:

Find the slant asymptotes, if any, of the following functions:

1. $f(x) = 4x^2-5x+10/ 2x+5$
2. $f(x) = x^2 - 6x + 3 / 3x$
3. $f(x) = 5x^2 - 5x - 5 / 5x$
4. $f(x) = (x+5)(x-4)/ 0.5x$



A **horizontal asymptote** is a horizontal line which the function never touches as it extends to negative or positive infinity. The horizontal asymptote is associated with a single y-value. For example, if an equation has a horizontal asymptote when y is 3, the equation for this line is $y=3$.

To find horizontal asymptotes of rational functions, consider the following rules:

1) If the **degree (the exponent of the leading coefficient) of the denominator is higher than the degree of the numerator**, the function has a horizontal asymptote that lies along the x-axis.

2) If the **degree of the denominator is smaller than the degree of the numerator**, the function will have no horizontal asymptotes.

3) If the **degrees of the numerator and the denominator are equal**, there will be a horizontal asymptote at the y-value determined by the quotient of these leading coefficients (divide the leading coefficient of the numerator by the leading coefficient of the denominator)

SAMPLE QUESTIONS:

Determine if these functions have a horizontal asymptote. If so, state the equation for the asymptote.

1. $f(x) = 3x^3/ (2x^4+4)$
2. $f(x) = 5x^4+4x^2 + 5/ (2x+3)$
3. $f(x) = 4x / 3x^3$
4. $f(x) = 3x^3 + 4x + 4/ ((2+x)(4x^2+3))$