MATH 312: ASSIGNMENT 1
DUE DATE: SEPTEMBER 21, 2012

(1) Prove the following stronger version of Dirichlet's approximation result. If α is a real number and n is a positive integer, then there are integers a and b such that $1 \leq a \leq n$ and $\mid a\alpha - b \mid \leq \frac{1}{n+1}$.

(2) Show that an infinite subset of a countable set is countable.

(3) Use mathematical induction to prove that

$$1^2 + 2^2 + \cdots + n^2 = \sum_{i=1}^{n} i^2 = \frac{1}{6} n(n+1)(2n+1).$$

(4) Show that if a is an integer, then $3 \mid a^3 - a$.

(5) Use mathematical induction to show that $n^5 - n$ is divisible by 5 for every positive integer $4n$.

(6) If a and b are integers not both zero, show that the greatest common divisor (gcd) (a, b) exists and

$$(a, b) = m_0 a + n_0 b$$

for some integers m_0 and n_0.

(7) Find the smallest prime between n^2 and $(n+1)^2$ for all positive integers n with $n \leq 10$.

(8) Use Dirichlet's theorem to show that there are infinitely many primes whose decimal expansion ends with a 1.

Date: September 14, 2012.