600D: ASSIGNMENT 1
DUE: OCT 26, 2011

i) Let R be a ring and P a finitely generated R-module. Show that there is a well-defined homomorphsim $Aut(P) \to K_1(R)$. Use this to show that there is a natural product operation $K_0(R) \otimes K_1(S) \to K_1(R \otimes S)$. ii) Show that a ring R and $M_n(R)$ are Morita equivalent and conclude that $K_i(R) \simeq K_i(M_n(R))$ for $i = 0, 1$. (Two rings R and S are said to be Morita equivalent if the categories $R - mod$ and $S - mod$ are equivalent). iii) Let (r, s) be a unimodular row over a commutative ring R. Define the *Mennicke symbol* $\begin{bmatrix} s \\ r \end{bmatrix}$ to be the class in $SK_1(R)$ of the matrix $\begin{pmatrix} r & s \\ t & u \end{pmatrix}$ where t, $u \in R$ satisfy $ru - st = 1$. Show that this symbol is independent of the choice of t and u, and that we have

$$\begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} s \\ r \end{bmatrix}, \quad \begin{bmatrix} s \\ r \end{bmatrix} \begin{bmatrix} s' \\ r \end{bmatrix} = \begin{bmatrix} ss' \\ r \end{bmatrix}.$$

iv) Consider the function $\rho_n : R^{n-1} \to St_n(R)$ sending $(r_1, \cdots, r_{n-1})$ to the product homomorphism $x_{in}(r_1)x_{2n}(r_2)\cdots x_{n-1,n}(r_{n-1}$. Show using the Steinberg relations that this is a group homomorphism. Show that ρ is an injection by showing that the composition $\phi\rho : R^{n-1} \to St_n(R) \to GL_n(R)$ is an injection.