Structure of Neutron Stars

The Nuclear Equation of State and Other Things

Giacconi et al. (1962)

- Rocket carried 3 Geiger counters to 225km - one counter failed (#1).
- Windows of counters pointed 55° from the axis of the rocket (i.e. the Zenith).
- Windows made of mica covered with lampblack.
- Rocket rotated at 2.0 rps, 350 s above 80km.

The Data

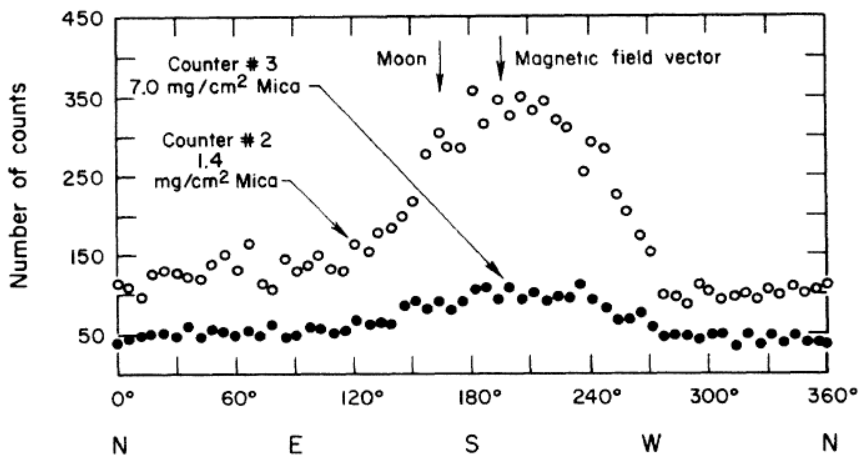
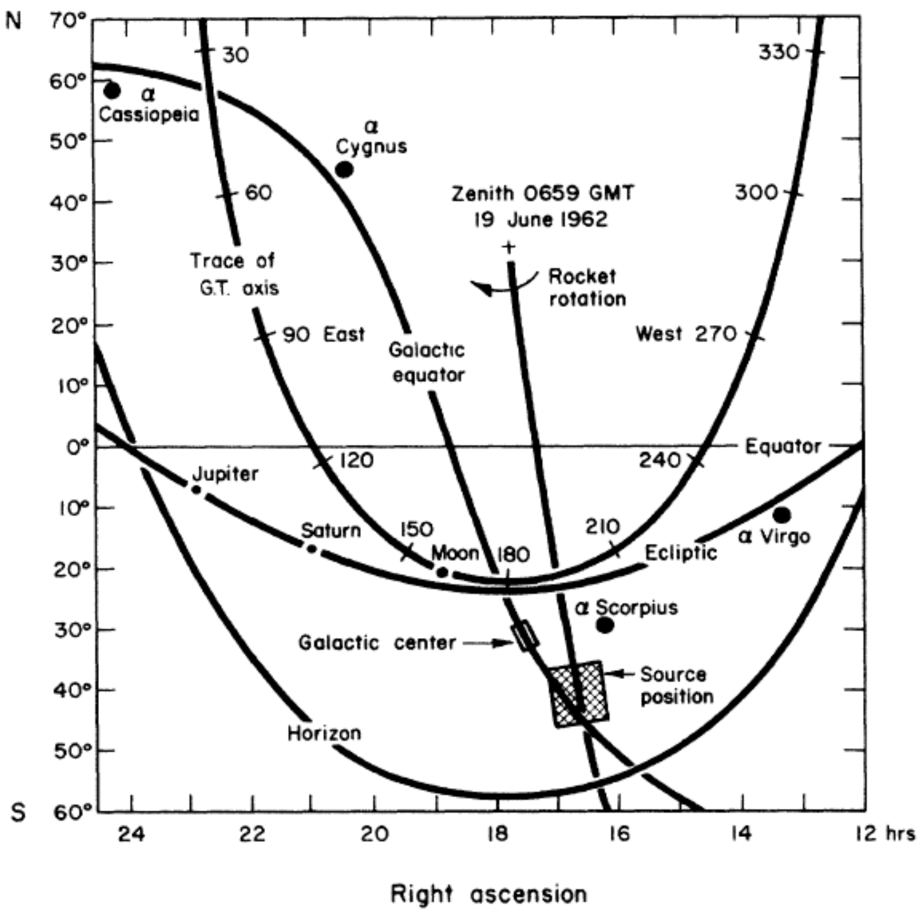


FIG. 1. Number of counts versus azimuth angle. The numbers represent counts accumulated in 350 seconds in each 6° angular interval.

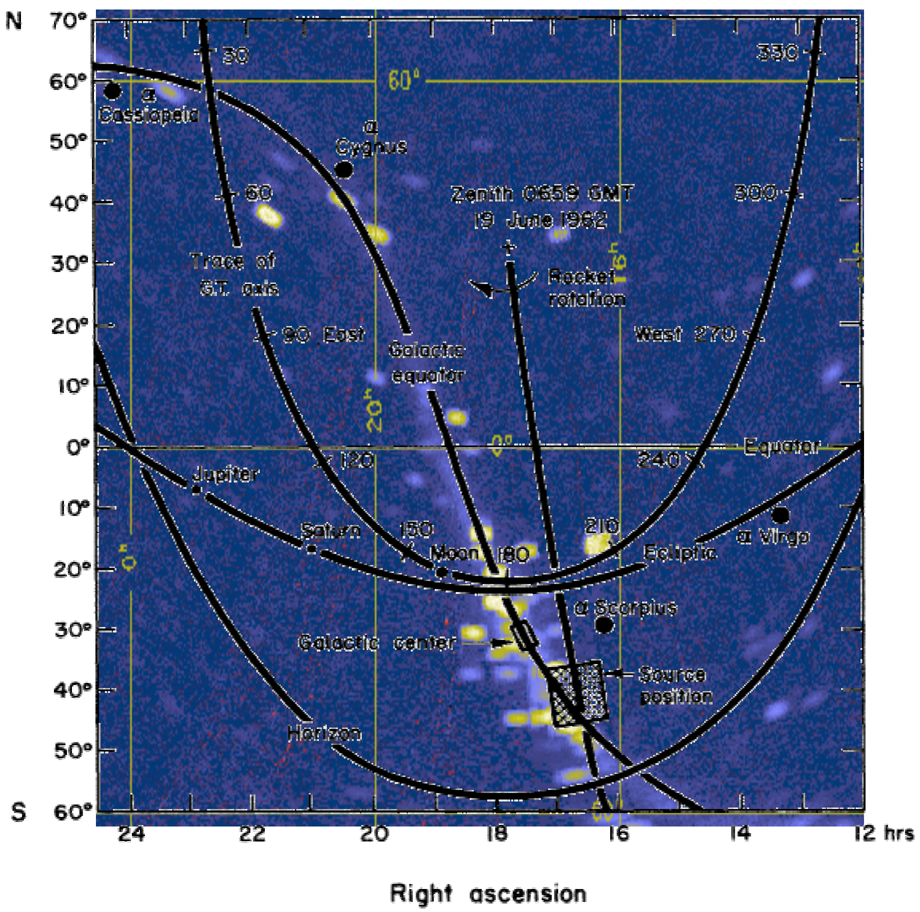


The Results

- They argued that the source was located about 10° above the horizon by comparing the absorption through the air and the mica assuming monochromatic source.

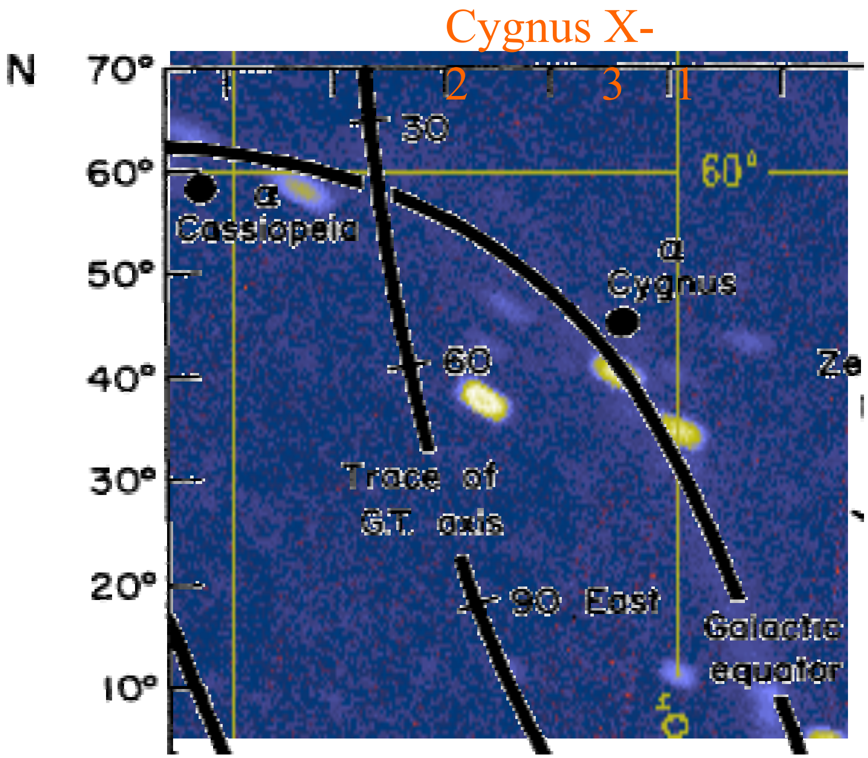
What Did the Aerobee See?

- Fig. 2 superimposed on the HEAO all-sky survey.
- The object known as Sco X-1 is the brightest in the X-ray sky at an azimuth angle of 210° along the G.T. axis.



What about the scruff at 60°?

- The detection at 60° albeit not as dramatic as the 210° result is also associated with an LMXB, Cygnus X-2.
- What is the flux of Cygnus X-2 compared with Sco X-1?



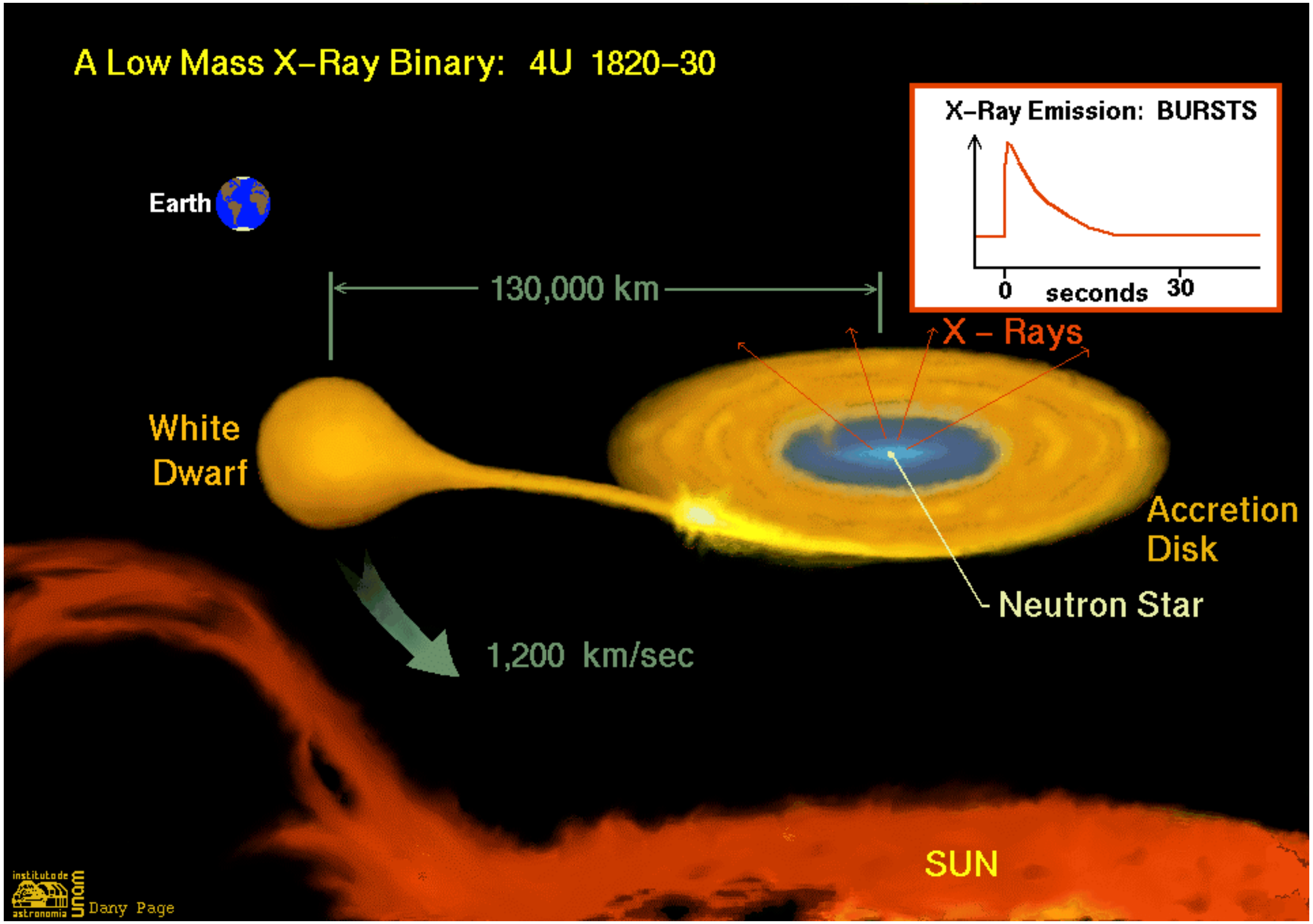
The Answer:

- Sco X-1: 14000 μJy (LMXB)
- Cyg X-1: 235-1320 μJy (HMXB)
- Cyg X-2: 450 μJy (LMXB)
- Cyg X-1: 90-430 μJy (HMXB)
- BTW:
  - What were they looking for from the moon?
  - What became of American Science and Engineering?

LMXBs and HMXBs

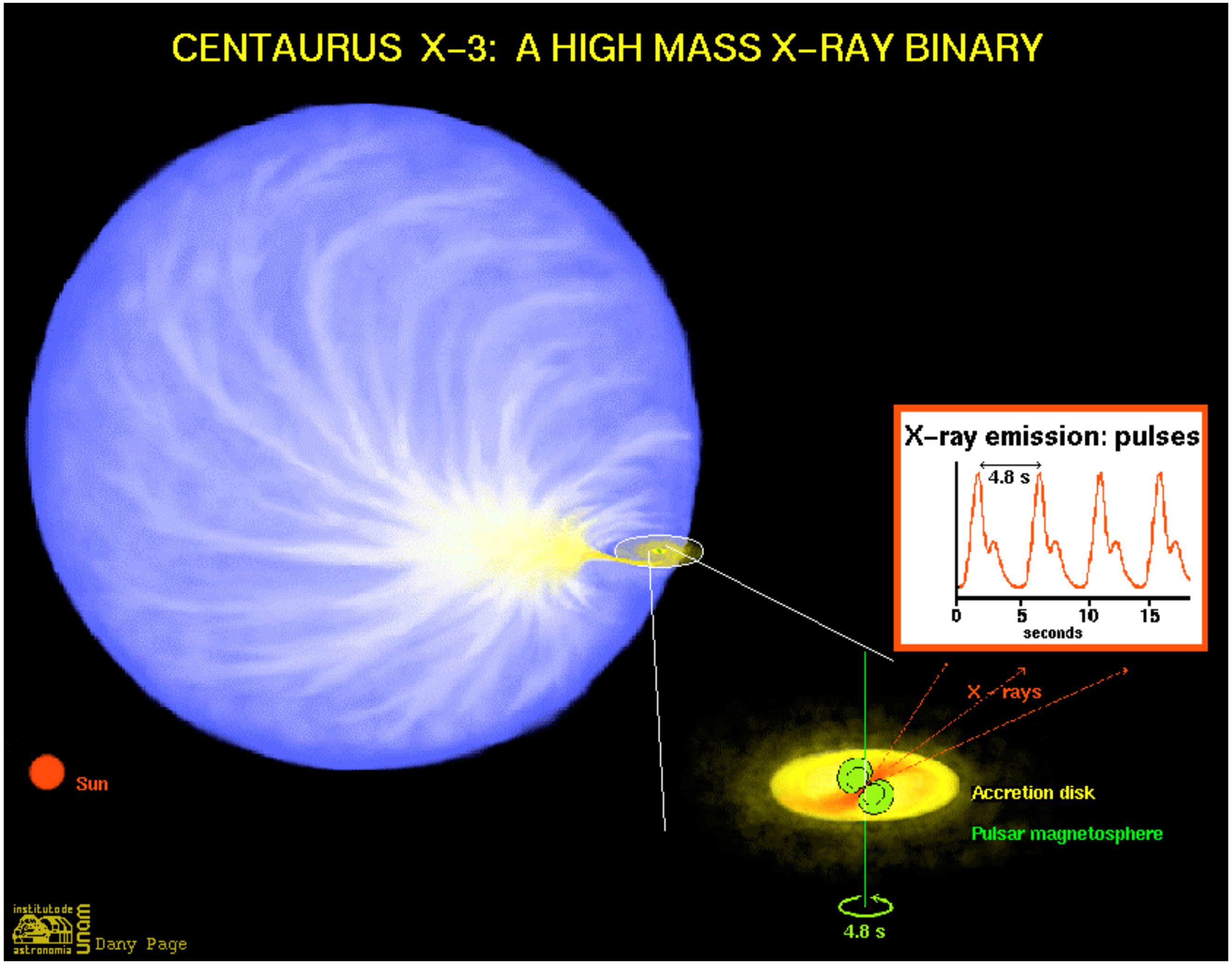
- Low-mass X-ray Binaries
  - Low mass main sequence star or white dwarf in orbit with a neutron star or black hole
  - Roche lobe overflow driven by gravitational radiation
- High-mass X-ray Binaries
  - High mass main sequence star in orbit with a neutron star or black hole
  - Wind accretor or Roche lobe overflow

LMXBs



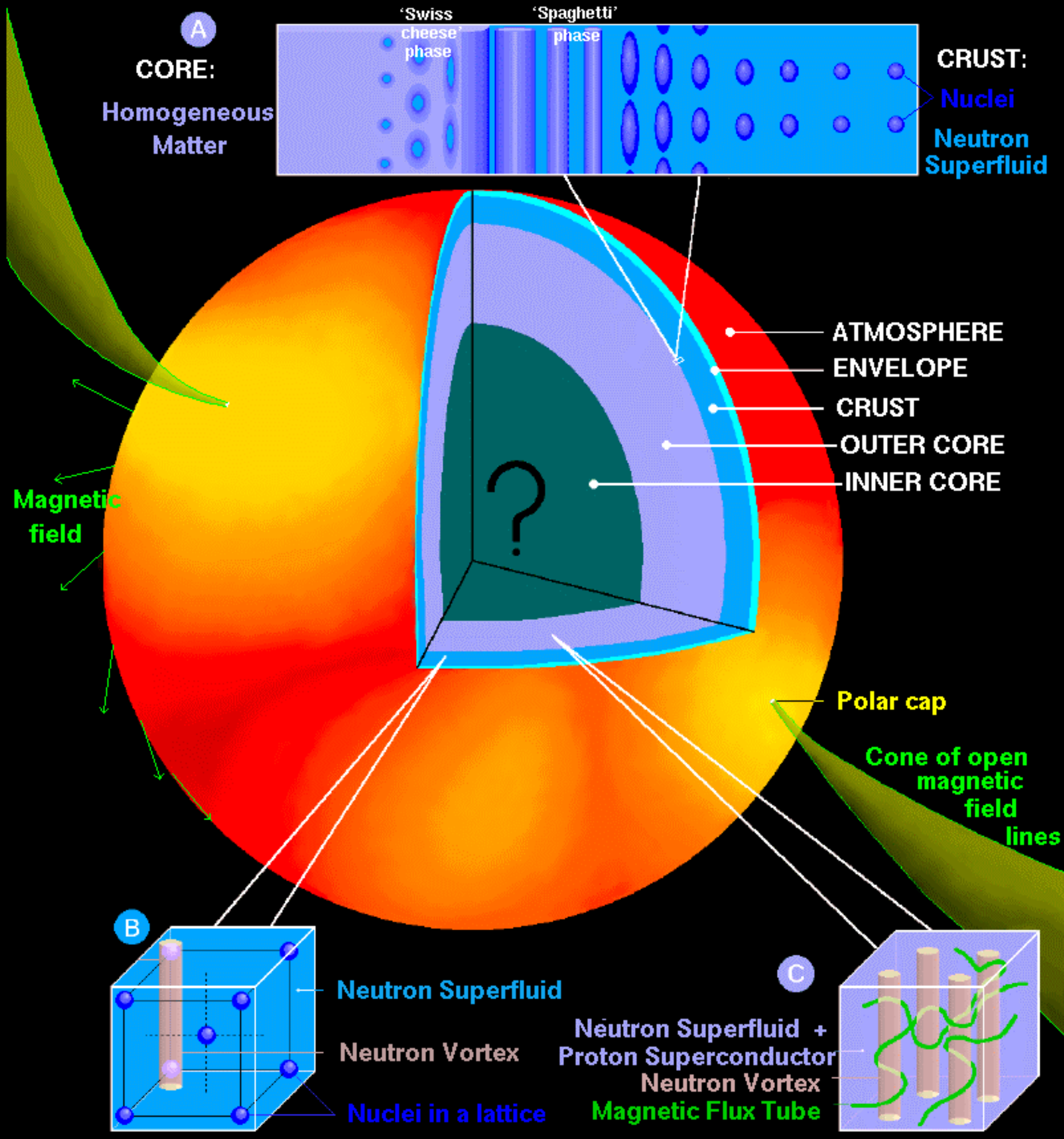
A Low Mass X-Ray Binary: 4U 1820-30
X-Ray Emission: BURSTS
0
seconds
30
Earth
130,000 km
X - Rays
White Dwarf
Accretion Disk
Neutron Star
1,200 km/sec
SUN
Dany Page

HMXBs



CENTAURUS X–3: A HIGH MASS X–RAY BINARY
X–ray emission: pulses
4.8 s
0
5
10
15
seconds
X – rays
Sun
Accretion disk
Pulsar magnetosphere
4.8 s
institutode astronomia unam
Dany Page

A NEUTRON STAR: SURFACE and INTERIOR



instituto de astronomía unam Dany Page

What are the various regions?

- Atmosphere: the region near the stellar surface where most of the photons originate.   Only a few millimeters thick.
- Envelope: the surface region that throttles the heat flux (more on this next week): free electrons and nuclei, a metal. (sometimes called outer crust)
- Crust: free electrons, nuclei and free neutrons (sometimes called inner crust)

More regions

- Outer core: free neutrons, free protons, free electrons and other particles (no more nuclei)
- Inner core: dunno. It could be like the outer core or it could contain free quarks.
- In a "quark" or "strange" star, the core and inner crust consist of free quarks.

Relativistic Stellar Structure (1) - Equations

- OV (1939) give the equations of stellar structure in GR.

$$\frac{du}{dr} = 4\pi r^2 \rho(p)$$

$$\frac{dp}{dr} = -\frac{p + \rho(p)}{r(r - 2u)} \left(4\pi p r^3 + u\right)$$

ρ	Energy Density
p	Pressure
u	Enclosed **gravitational** mass
r	Circumferential radius

Relativistic Stellar Structure (2) - What's new?

- The relativistic equations of stellar structure are deceptively similar to the Newtonian results.

$$\frac{du}{dr} = 4\pi r^2 \rho(p)$$

$$\frac{dp}{dr} = -\frac{\boxed{p} + \rho(p)}{r(r \boxed{- 2u})} \left(\boxed{4\pi p r^3} + u \right)$$

New Bits

Relativistic Stellar Structure (3) - Nonlinearity

- Let's take $\rho \rightarrow \alpha\rho$ and see how the equations transform.

$$\frac{du}{dr} = 4\pi r^2 \alpha\rho(p) \quad \text{so} \quad u \rightarrow \alpha u \qquad \boxed{\text{New Bits}}$$

$$\frac{dp}{dr} = -\frac{\boxed{p} + \alpha\rho(p)}{r(r \boxed{- 2\alpha u})}\left(\boxed{4\pi p r^3} + \alpha u\right)$$

- If it weren't for the new bits, we would have $p \rightarrow \alpha^2 p$, but the pressure generates more gravity. Even worse so does the gravity.

Relativistic Stellar Structure (4) - Nonlinearity

- The nonlinearity in the pressure is sufficient to transform a well-behaved solution into a singular one.
- The term in the denominator is even less benign. It defines a radius where the gravitational acceleration diverges.

$$\frac{du}{dr} = 4\pi r^2 \alpha\rho(p) \quad \text{so} \quad u \rightarrow \alpha u$$

$$\frac{dp}{dr} = -\frac{p + \alpha\rho(p)}{r(r - 2\alpha u)}\left(4\pi p r^3 + \alpha u\right)$$

New Bits

Relativistic Stellar Structure (5) - Solutions

- These nonlinear equations have a few non-trivial solutions, and one of them is pretty trivial.

$$\frac{du}{dr} = 4\pi r^2 \rho(p)$$
$$\frac{dp}{dr} = -\frac{p+\rho(p)}{r(r-2u)}\left(4\pi p r^3 + u\right)$$

$p = -\rho$ — The change in the pressure vanishes.

ρ is constant. — You will do this one.

Another Ingredient

- The equation of state is a relationship between the pressure and density of a material.
- Some examples:

$$p = \frac{1}{\mu m_u} \rho k T \qquad \text{classical ideal gas}$$

$$p = A \rho^{c_p/c_v} \quad \text{isentropic equation of state}$$

- Neutron stars are effectively cold so these equations of state don’t cut it.

Degeneracy Pressure (6)

- In astrophysics, we have two important regimes:
  - Electron supply the pressure and nuclei supply the mass:

$$p \propto \begin{cases} \rho^{5/3} & \text{NR} \\ \rho^{4/3} & \text{UR} \end{cases}$$

  - Neutrons supply the pressure and the mass

$$p \propto \begin{cases} \rho^{5/3} & \text{Non-Relativistic} \\ \rho & \text{Ultra-Relativistic} \end{cases}$$

Fermi Gases in Equilibrium

- In general there are several species in chemical equilibrium: nuclei, neutrons, proton, electrons (and other leptons). An example:

$$n \rightleftharpoons p + e^- + \bar{\nu}_e$$

$$\mu_n = \mu_p + \mu_e + \mu_{\bar{\nu}_e}$$

- For a non-degenerate species μ is essentially the mass of the particle, so if $\mu_e > m_n - m_p$ each new electron added to the gas combines with a proton to make a neutron until nearly all the protons are exhausted.
- For a degenerate species μ is the Fermi energy.

A basic neutron star core (1)

- In the core of a neutron star, you have neutrons, protons and electrons in equilibrium, so $\mu_n = \mu_p + \mu_e$

$$m_n(1+x_n^2)^{1/2} = m_e(1+x_e^2)^{1/2} + m_p(1+x_p^2)^{1/2}$$

- There is also charge balance, $\frac{1}{3\pi^2\lambda_e^3}x_e^3 = \frac{1}{3\pi^2\lambda_p^3}x_p^3$. Therefore $m_e x_e = m_p x_p$.

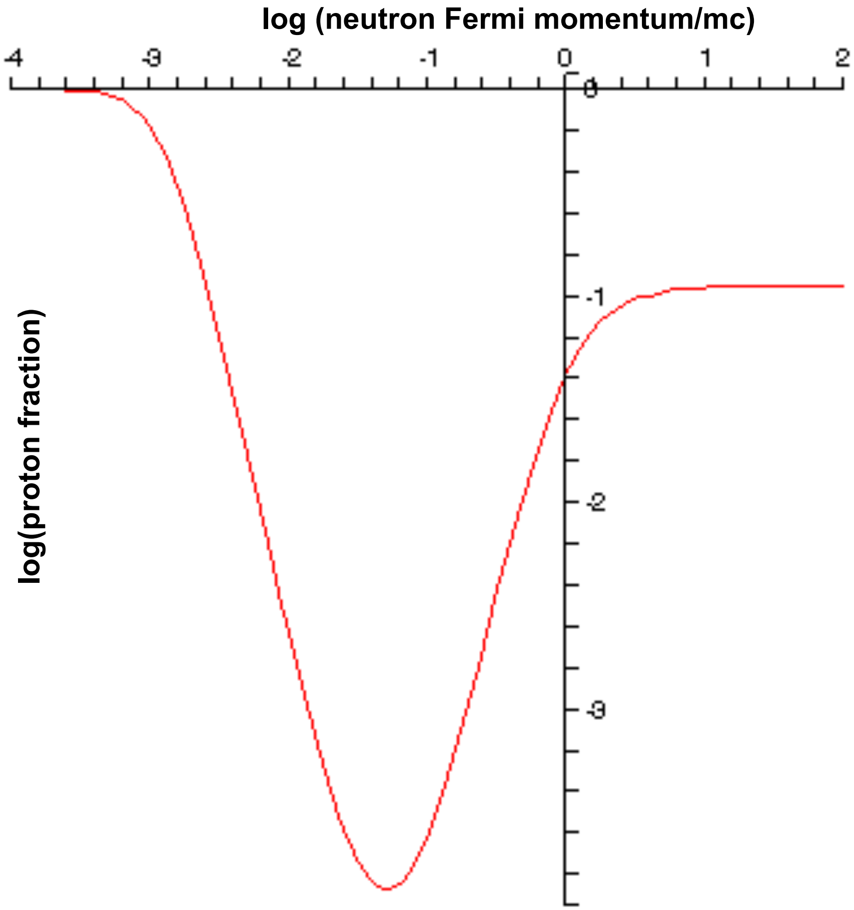
A basic neutron star core (2)

- Let's eliminate the electrons from the eqn $$m_n(1+x_n^2)^{1/2} = (m_e^2 + m_p^2 x_p^2)^{1/2} + m_p(1+x_p^2)^{1/2}$$ and solve for the ratio of protons to neutrons,

$$\frac{n_p}{n_n} = \left(\frac{m_p x_p}{m_n x_n}\right)^3$$
$$= \frac{1}{8} \frac{1}{(1+x_n^2)^{3/2} x_n^3 m_n^6} \Big\{ [(m_e - m_p)^2 - m_n^2(1+x_n^2)] \times$$
$$[(m_e + m_p)^2 - m_n^2(1+x_n^2)] \Big\}^{3/2}$$

A basic neutron star core (3)

- The neutrons appear at a finite density below which there are only protons and electrons, reach a maximum fraction and asymptote to 8/9 of the baryons.
- Sum over the different particles to get the total pressure.



Neutron and Quark Stars