Find where can polynomial long division be useful in other problems

By using the polynomial long division, we can solve equations that involve more than one variable in a less complex way.

For example:

$$\frac{x^3 + y^3 + z^3 - 3\,x\,y\,z}{x + y + z}$$

Just the equation itself already looks pretty intimidating, but with long division, we are able to figure out the quotient/solution and the remainder after the polynomial is divided by another polynomial.

First, we put it into long division form with (x+y+z) on the left outside the division form and ((x^3)+(y^3)+(z^3)-(3xyz)) on the inside of the division.

```
            ____________________________
x + y + z ) x^3 + y^3 + z^3 - 3xyz
```

Next, we divide x from the divisor into x^3, which is the leading term of the dividend, ((x^3)+(y^3)+(z^3)-(3xyz)).

```
            x^2
            _______________________________________
x + y + z ) x^3 + y^3 + z^3 - 3xyz
            x^3                     + x^2y + x^2z
            -------------------------------------
                  y^3 + z^3 - 3xyz - x^2y - x^2z
```

Third step, we'd divide y from the divisor into y^3.

```
            x^2 + y^2
            ______________________________________________________
x + y + z ) x^3 + y^3 + z^3 - 3xyz
            x^3                     + x^2y + x^2z
            -------------------------------------
                  y^3 + z^3 - 3xyz - x^2y - x^2z
                  y^3                               + xy^2 + y^2z
                  -------------------------------------------------
                        z^3 - 3xyz - x^2y - x^2z - xy^2 - y^2z
```

Step 4, divide z from the divisor into z^3.

```
            x^2 + y^2 + z^2
            ___________________________________________________________________
x + y + z ) x^3 + y^3 + z^3 - 3xyz
            x^3                     + x^2y + x^2z
            -------------------------------------
                  y^3 + z^3 - 3xyz - x^2y - x^2z
                  y^3                               + xy^2 + y^2z
                  -------------------------------------------------
                        z^3 - 3xyz - x^2y - x^2z - xy^2 - y^2z
                        z^3                                      + xz^2 + yz^2
                        ------------------------------------------------------
                              -3xyz - x^2y - x^2z - xy^2 - y^2z - xz^2 - yz^2
```

Step 5, rearrange to put the higher powers first

$$
\begin{array}{l}
\qquad\qquad\quad x^2 + y^2 + z^2 \\
\hline
x + y + z \;)\; x^3 + y^3 + z^3 - 3xyz \\
\qquad\qquad\quad x^3 + x^2y + x^2z \\
\hline
\qquad\qquad\quad y^3 + z^3 - 3xyz - x^2y - x^2z \\
\qquad\qquad\quad y^3 + xy^2 + y^2z \\
\hline
\qquad\qquad\quad z^3 - 3xyz - x^2y - x^2z - xy^2 - y^2z \\
\qquad\qquad\quad z^3 + xz^2 + yz^2 \\
\hline
\qquad\qquad\quad -x^2y - x^2z - xy^2 - y^2z - xz^2 - yz^2 - 3xyz
\end{array}
$$

Step 6, you have two terms with x^2 left. You can divide x into –x^y – x^2.

$$
\begin{array}{l}
\qquad\qquad\quad x^2 + y^2 + z^2 - xy - xz \\
\hline
x + y + z \;)\; x^3 + y^3 + z^3 - 3xyz \\
\qquad\qquad\quad x^3 + x^2y + x^2z \\
\hline
\qquad\qquad\quad y^3 + z^3 - 3xyz - x^2y - x^2z \\
\qquad\qquad\quad y^3 + xy^2 + y^2z \\
\hline
\qquad\qquad\quad z^3 - 3xyz - x^2y - x^2z - xy^2 - y^2z \\
\qquad\qquad\quad z^3 + xz^2 + yz^2 \\
\hline
\qquad\qquad\quad -x^2y - x^2z - xy^2 - y^2z - xz^2 - yz^2 - 3xyz \\
\qquad\qquad\quad -x^2y - x^2z - xy^2 - xz^2 - 2xyz \\
\hline
\qquad\qquad\quad - y^2z - yz^2 - xyz
\end{array}
$$

Step 7, divide y into –y^2z because you only have one term with y^2 left.

$$
\begin{array}{l}
\qquad\qquad\quad x^2 + y^2 + z^2 - xy - xz - yz \\
\hline
x + y + z \;)\; x^3 + y^3 + z^3 - 3xyz \\
\qquad\qquad\quad x^3 + x^2y + x^2z \\
\hline
\qquad\qquad\quad y^3 + z^3 - 3xyz - x^2y - x^2z \\
\qquad\qquad\quad y^3 + xy^2 + y^2z \\
\hline
\qquad\qquad\quad z^3 - 3xyz - x^2y - x^2z - xy^2 - y^2z \\
\qquad\qquad\quad z^3 + xz^2 + yz^2 \\
\hline
\qquad\qquad\quad -x^2y - x^2z - xy^2 - y^2z - xz^2 - yz^2 - 3xyz \\
\qquad\qquad\quad -x^2y - x^2z - xy^2 - xz^2 - 2xyz \\
\hline
\qquad\qquad\quad -y^2z - yz^2 - xyz \\
\qquad\qquad\quad -y^2z - yz^2 - xyz \\
\hline
\qquad\qquad\quad 0
\end{array}
$$

Polynomial long division can also be helpful in trying to solve problems involving square roots.

In order to solve:

$$\frac{x^2 - \sqrt{3}}{\sqrt{2}\ x^2 + 1}$$

Divide the leading term x^2 of the numerator polynomial by the learning term sqrt(2)x^2 of the divisor:

$$\begin{array}{r|ll} & \frac{1}{\sqrt{2}} & \\ \hline \sqrt{2}\,x^2 + 1 & x^2 & -\sqrt{3} \end{array}$$

Multiply "back": $\frac{1}{\sqrt{2}}\left(\sqrt{2}\,x^2 + 1\right) = x^2 + \frac{1}{\sqrt{2}}$, and subtract

Next, you divide the leading term x^2 of the numerator polynomial by the leading term sqrt(2)x^2 of the divisor:

$$\begin{array}{r|ll} & \frac{1}{\sqrt{2}} & \\ \hline \sqrt{2}\,x^2 + 1 & x^2 & -\sqrt{3} \\ & x^2 & +\frac{1}{\sqrt{2}} \\ \hline & & -\left(\sqrt{3} + \frac{1}{\sqrt{2}}\right) \end{array}$$

Therefore, you answer would be:

$$\frac{x^2 - \sqrt{3}}{\sqrt{2}\,x^2 + 1} = \frac{1}{\sqrt{2}} - \frac{\sqrt{3} + \frac{1}{\sqrt{2}}}{\sqrt{2}\,x^2 + 1}$$

$\frac{1}{\sqrt{2}}$ being the divisor of the polynomial long division

$-\frac{\sqrt{3} + \frac{1}{\sqrt{2}}}{\sqrt{2}\,x^2 + 1}$ being the remainder of the polynomial long division.