# ASSIGNMENT 4

DUE: OCT 27, 2011

(I) (15 points)

a)Show that the order of the product of two disjoint cycles of lengths $m$ and $n$ is lcm(m,n) where lcm=least common multiple.
b) What is the order of the product of $k$-disjoint cycles of length $(m_1, \cdots, m_k)$.
c)How will you find the order of a given permutation in $S_n$?

(2)(10 points)

Let $G$ be a finite group of order $pq$, where $p > q$ are primes.
a)Show that $G$ has a subgroup of order $p$ and a subgroup of order $q$.
b) Given two primes $p, q$ such that $q$ divides $p - 1$, show that there exists a non-abelian group of order $pq$.

(3)(10 points)

a) Given $\alpha = (1, 2)(3, 4)$ in $S_6$ and $\beta = (5, 6)(1, 3)$ , show that $\alpha$ and $\beta$ are conjugate. Find an element of $S_6$ that conjugates them.
b) Are $(1, 2)(4, 5)$ and $(2, 1, 4, 5)$ conjugate in $S_6$? Justify your answer.

(4)(15 points)

a) Can $S_4$ have conjugacy classes of sizes 1, 3, 6, 6, 8? What about 2, 4, 8 ,5, 5?
b) In $S_7$, express $(1, 2)(1, 2, 3)(1, 2)$ as a product of disjoint cycles and write its cycle type.
c) Prove that $(1, 2, \cdots, n)^{-1} = (n, n - 1, n - 2, \cdots, 2, 1)$.