ASSIGNMENT 1

DUE: SEPTEMBER 22, 2011

1 (10 points)
  (a) Let $A = \{1, 2, ..., 8\}$ Define a relation R on A by $(a, b) \in R$ if and only if $3|a - b$ for all $a, b \in A$. Show that R is an equivalence relation. Find the equivalence classes $[1], [2], [3], [4]$.
  (b) In $\mathbb{Z}_6$, which of the following equivalence classes are equal? [-1] ,[2] ,[8] ,[5] ,[-2] ,[11] ,[23].

2 (10 points)
Let $f : A \to B, g : B \to C$ be functions, recall that for each $A' \subseteq A, B' \subseteq B$,we define

$$f(A') = \{f(a) : a \in A'\} \subseteq B,\ f^{-1}(B') = \{a : f(a) \in B'\} \subseteq A$$

  (a) For each $X, Y \subseteq B$ and $Z, W \subseteq A$, Show that

$$f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$$
$$f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$$
$$f(Z \cup W) = f(Z) \cup f(W)$$
$$f(Z \cap W) \subseteq f(Z) \cap f(W)$$

  (b) Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ and $Z, W \subseteq \mathbb{R}$ such that $f(Z \cap W) \neq f(Z) \cap f(W)$.
  (c) Find an example of a map f, g such that $g \circ f$ is onto but f is not onto. Find an example of a map f, g such that $g \circ f$ is 1-1 but g is not 1-1.

3 (10 points)
  (a) Let $G = \mathbb{Q}\backslash\{-1\}$. Show that $(G, *)$ is a group where $a * b = a + b + ab$. (Don't forget to check that $*$ is a binary operation on G i.e. $a \in G, b \in G \Rightarrow a * b \in G$.)
  (b) Consider $(\mathbb{Z}/12, *)$ where $[a] * [b] = [ab]$. Is this a group?
  (c) Consider the group of 2×2 matrices over $\mathbb{R}$ of determinant 1. Is it a subgroup of the group of 2×2 matrices over $\mathbb{R}$ of nonzero determinant ?