[12] **2.** (a) [3] Find $\frac{d}{dx}\left(\frac{x^4+x^{7/2}}{x^2}\right)$. Remember (see the instructions above Question 1) that your answers must be completely simplified in Questions 1 and 2.

$$\frac{x^4+x^{7/2}}{x^2} = x^6 + x^{7/2+2}$$

Answer $6x^5 + \left(\frac{7}{2}+2\right)x^{7/2+2-1}$

(b) [3] If $y = x^2\cos x$, find the second derivative y''. Express your answer in the form $p(x)\sin x + q(x)\cos x$ where $p(x)$ and $q(x)$ are polynomials.

Answer

$$y' = x^2(-\sin(x)) + 2x(\cos(x))$$

$$y' = 2x\cos(x) + 2\cos(x)$$

(c) [3] If $y = x\sin(\sqrt{x}+x)$, find y'.

Answer

$$y' = x\sin(\sqrt{x}+x)\left(\tfrac{1}{2}x^{-1/2}+1\right) + \sin(\sqrt{x}+x)$$

(d) [3] f is a function that satisfies $f'(e) = e$. Let $g(x) = f(e^{x^2})$. Find $g'(1)$.

Answer $g'(1) = e$

$$g'(x) = f'(e^{x^2})e^{x^2-1}$$

$$g'(1) = f'(e^1)e^{1-1}$$

$$= e \cdot e^0 = e$$

Continued on page 4

[12] **2.** (a) [3] Find $\frac{d}{dx}\left(\frac{x^4+x^{7/2}}{x^2}\right)$. Remember (see the instructions above Question 1) that your answers must be completely simplified in Questions 1 and 2.

$$\frac{x^4+x^{7/2}}{x^2} = x^6 + x^{7/2+2}$$

Answer $6x^5 + \left(\frac{7}{2}+2\right)x^{7/2+2-1}$

(b) [3] If $y = x^2\cos x$, find the second derivative y''. Express your answer in the form $p(x)\sin x + q(x)\cos x$ where $p(x)$ and $q(x)$ are polynomials.

$$y' = x^2(-\sin(x)) + 2x(\cos(x))$$

$$y' = 2x\cos(x) + 2\cos(x)$$

Answer

(c) [3] If $y = x\sin(\sqrt{x}+x)$, find y'.

$$y' = x\sin(\sqrt{x}+x)\left(\tfrac{1}{2}x^{-1/2}+1\right) + \sin(\sqrt{x}+x)$$

Answer

(d) [3] f is a function that satisfies $f'(e) = e$. Let $g(x) = f(e^{x^2})$. Find $g'(1)$.

$$g'(x) = f'(e^{x^2})e^{x^2-1}$$

$$g'(1) = f'(e^1)e^{1-1}$$

$$= e \cdot e^0 = e$$

Answer $g'(1) = e$

Continued on page 4

Full-Solution Problems. In questions 3–7, justify your answers and show all your work. If a box is provided, write your final answer there. **Unless otherwise indicated, simplification of answers is not required in these questions.**

[7] **3.** Let $f(x) = \dfrac{\sqrt{x^2+1}}{3x+5}$.

(a) [4] Determine the horizontal asymptotes of the graph $y = f(x)$.

Answer

horizontal asymptotes occur when denominator is zero

$3x+5=0$

$\Rightarrow x = -5/3$ is a horizontal asymptote

(b) [3] Determine the vertical asymptote(s) of the graph $y = f(x)$. For each vertical asymptote $x = a$, determine whether each of the one-sided limits "equals" ∞ or $-\infty$ as x approaches a.

Need to find $\lim\limits_{x\to\infty} \dfrac{\sqrt{x^2+1}}{3x+5}$

L'hôpital says $= \dfrac{\frac{1}{2}(x^2+1)2x}{3}$

$= \infty$

no ~~A~~ vertical asymptotes!

Since $\lim\limits_{x\to-\infty} = -\infty$ too.

Continued on page 5

[6] 4. Let a and b be constants, and define

$$f(x) = \begin{cases} (x^2 + b) & \text{if } x < 1 \\ ax + b & \text{if } 1 \le x \le 2 \\ 5x - 3 & \text{if } x > 2 \end{cases}$$

Find the values of a and b for which f is continuous at $x = 1$ and $x = 2$. Fully justify your answer.

Answer
$a = 2, b = 3$

WANT $(x^2 + b) = ax + b$ at $x = 1$

DERIVE $2x + 0 = a$

$\Rightarrow \boxed{a = 2}$

WANT $ax + b = 5x - 3$ at $x = 2$

$2x + b = 5x - 3$

$2 \cdot 2 + b = 5 \cdot 2 - 3$

$b = 10 - 3 - 4$

$\boxed{b = 3}$

Continued on page 6

Name:

[4] **5.** Prove that the equation $x^3 - 3x + 1 = 0$ has at least two positive real solutions. Carefully cite any theorem you use, and justify why the theorem can be used.

IVT says that if a function f is cont then there is a root

So, by IVT $y = x^3 - 3x + 1$ is cont so we can use it

$f(-2) = (-2)^3 - 3(-2) + 1 = -8 + 6 + 1 =$

$f(-1) = (-1)^3 - 3(-1) + 1 = -1 + 3 + 1 =$

$f(0) = (0)^3 - 3(0) + 1 = 1$

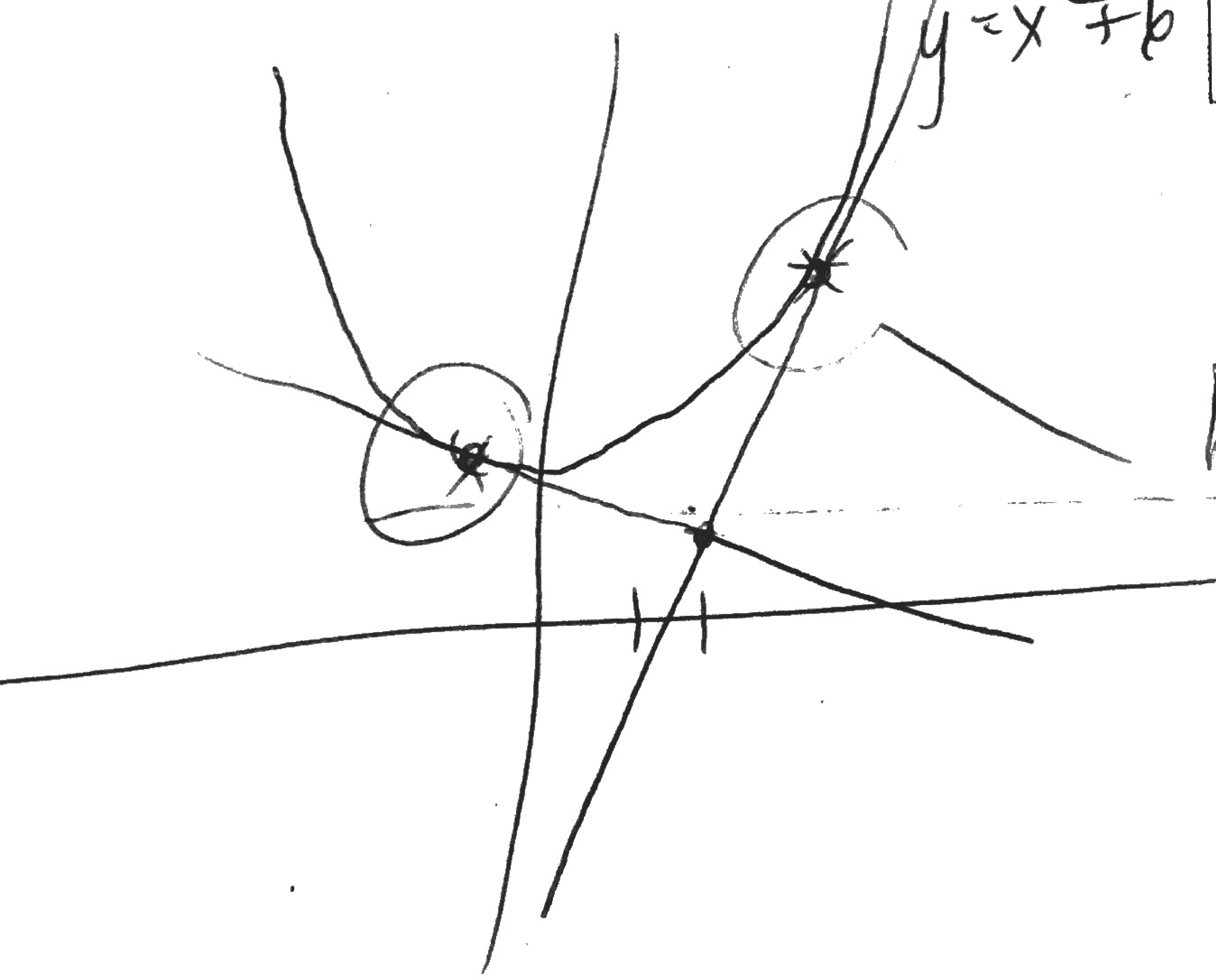
$f(1) = (1)^3 - 3(1) + 1 = -1$

So f has 2 roots.

[6] **6.** Find the point(s) on the curve $y = x^2 + 6$ such that the tangent line(s) to the curve at these point(s) pass through the point $(2, 1)$.

Answer

$y = x^2 + 6$



here they are.

Continued on page 7

Name:

[6] 7. Use the definition of the derivative in the questions below. No marks will be given for any other method. In this and any other question, you may use the back of a page if necessary, but please indicate so.

(a) [3] Find

$$\frac{d}{dx}\left(\frac{1}{1-x}\right)$$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\left(\frac{1}{1-(x+h)} - \frac{1}{1+x}\right)}{h}$$

common denominator $-(1-x)^{-2}(-1)$

$= (1-x)^{-2}$

(b) [3] Determine whether the function

$$f(x) = \begin{cases} x^3 \sin\frac{1}{x} & \text{if } x < 0 \\ 4x^2 - x & \text{if } x \geq 0 \end{cases}$$

is differentiable at $x = 0$.

$f(0) = 4(0)^2 - 0 = 0$

$f(-0.0001) = (-0.0001)^3 \sin(\infty)$

as $0.0001 \to 0$

$\Rightarrow$

$f(0)$ is a discontinuity

$\Rightarrow f$ is not diff at 0

The End