MATH 312: ASSIGNMENT 4

YOU MAY TURN IN THIS ASSIGNMENT IN TWO INSTALMENTS, ONE DUE ON NOV 17 and THE OTHER ON NOV 24.

1) Find the least nonnegative residue modulo 28 of 12,345 and -54321.

2) Find the least positive residue of $1! + 2! + 3! + \cdots + 100!$ modulo 12 and 25.

3) Show that if a, b and c are integers with $c > 0$, such that $a \equiv b \bmod c$, then $(a, c) = (b, c)$.

4) Show that if $aj \equiv b_j \bmod m$ for $j = 1,\ 2, \cdots m$ where m is a positive integer, then
the products $a_1.a_2 \cdots .a_j$ and $b_1.b_2 \cdots .b_j$ are congruent modulo m.

5) Show by mathematical induction that if n is a positive integer, then $5^n \equiv\ 1 + 4n \bmod 16$.

6) Find the least positive residue of 16! mod 17 and $3^1 0$ modulo 11.

7) find all solutions of $2x + 4y \equiv\ 6$ modulo 8.

8) Find an integer that leaves a remainder of 2 when divided by either 3 or 5, but that is divisible by 4.

9) What is the multiplicative inverse of 5 modulo 17?

10) Solve the following simultaneous system of congruences:
$x \equiv\ 4 \bmod 6, x \equiv\ 13 \,\mathrm{mod} 15$.

11) What is the highest power of 5 that divides 235,555,790 and the highest power of 2 that divides 89,375,744?

12) Is 1086320015 divisible by 11?

13) Which of 13,19,21 and 27 divide 2340?

14) Using the check digit system described for passports, determine the check digit that should be added to 132999.

15) Show that $1^{p-1} + 2^{p-1} + \cdot + (p-1)^{p-1} \equiv\ -1$ mod p.

16) Show that if n is an odd composite integer pseudoprime to the base a, then n is a pseudoprime to the base $n - a$.

17) Use the Pollard method to find a divisor of 7,331,117. (You may use a computer).

18) Show that 1387 is a pseudoprime, but not a strong pseudoprimt to the base 2.

19) Check (by factoring and using the criterion) hat 321,197,185 is a Carmichael number.

20) Find a reduced residue system modul0 14.