ASSIGNMENT 3

DUE: OCTOBER 13, 2011

1 (10 points) For a subgroup H of G define the left coset aH of H in G as the set of all elements of the form $ah,\ h \in H$. The right coset Ha is the set of all elements of the form $ha\ h \in H$. Show that there is a one-to-one correspondence between the set of left cosets of H in G and the set of right cosets of H in G.

2 (10 points) Suppose that H is a subgroup of G such that whenever $Ha \neq Hb$ then $aH \neq bH$. Prove that $gHg^{-1} \subset H$ for all $g \in G$.

3 (10 points) Let G be a finite group whose order is not divisible by 3. Suupose that $(ab)^3 = a^3b^3$. Prove that G must be abelian.

4 (10 points) If N is normal in G and $a \in G$ is of order $o(a)$, prove that the order of aN in G/N is a divisor of $o(a)$.

5 (10 points) Let G be the group of nonzero complex numbers under multiplication and let $\bar{G}$ be the group of all real (2×2) matrices of the form $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$, where not both a and b are 0, under matrix multiplication. Show that G and $\bar{G}$ are isomorphic. (Hint: Represent a complex number as $(a + ib)$ where $a,\ b$ are real).