Periodic task scheduling

Static priority scheduling
Rate monotonic priority assignment
Derivation of the RM utilization bound

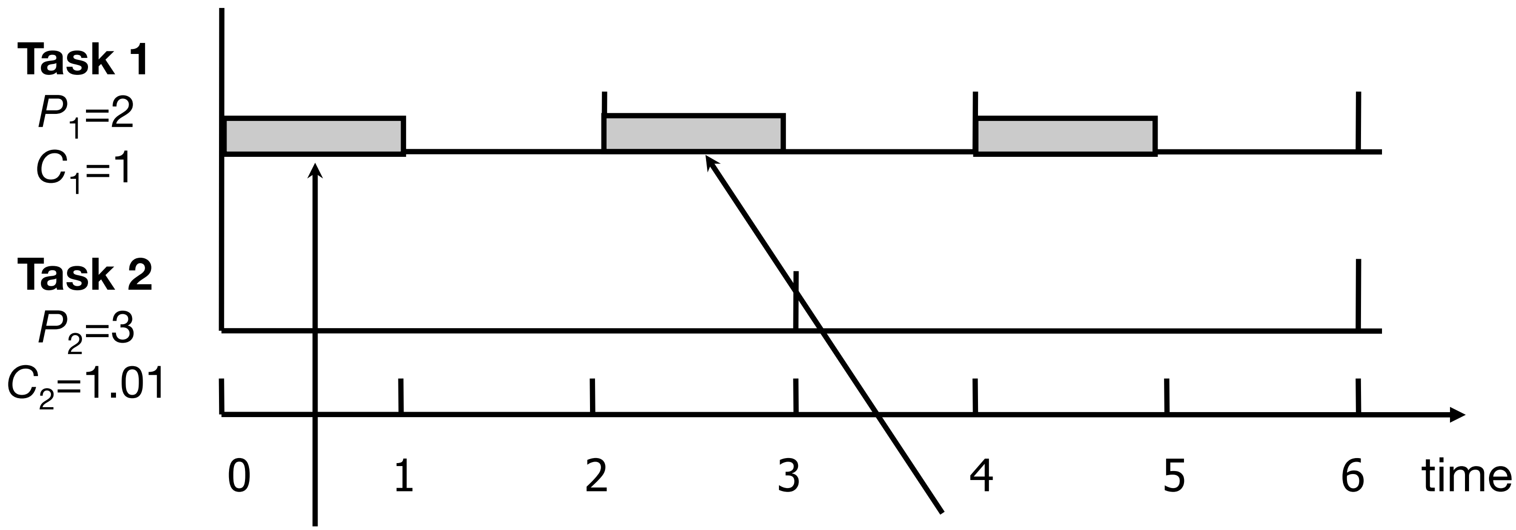
Impact of GRMS

- GRMS: Generalized Rate Monotonic Scheduling
- Cited in the Selected Accomplishment section of the National Research Council's report on A Broader Agenda for Computer Science and Engineering in 1992.
- "Through the development of Rate Monotonic Scheduling [theory], we now have a system that will allow [Space Station] Freedom's computers to budget their time, to choose between a variety of tasks, and decide not only which one to do first but how much time to spend in the process." [Deputy Administrator of NASA, Aaron Cohen]
- "The navigation payload software for the next block of Global Positioning System upgrade recently completed testing. ... This design would have been difficult or impossible prior to the development of rate monotonic theory." [L. Doyle, and J. Elzey, "Successful Use of Rate Monotonic Theory on A Formidable Real-Time System"]

Review

- Terminology
  - Definitions of tasks, task invocations, release/arrival time, absolute deadline, relative deadline, period, start time, finish time, …
  - Preemptive versus non-preemptive scheduling
  - Priority-based scheduling
  - Static versus dynamic priorities
- Utilization (U) and schedulability
  - Main problem: Find *Bound* for scheduling policy such that
    - *U* < *Bound* → All deadlines met!
- Optimality of EDF scheduling
  - $Bound_{EDF}$ = 100%

A quick refresher



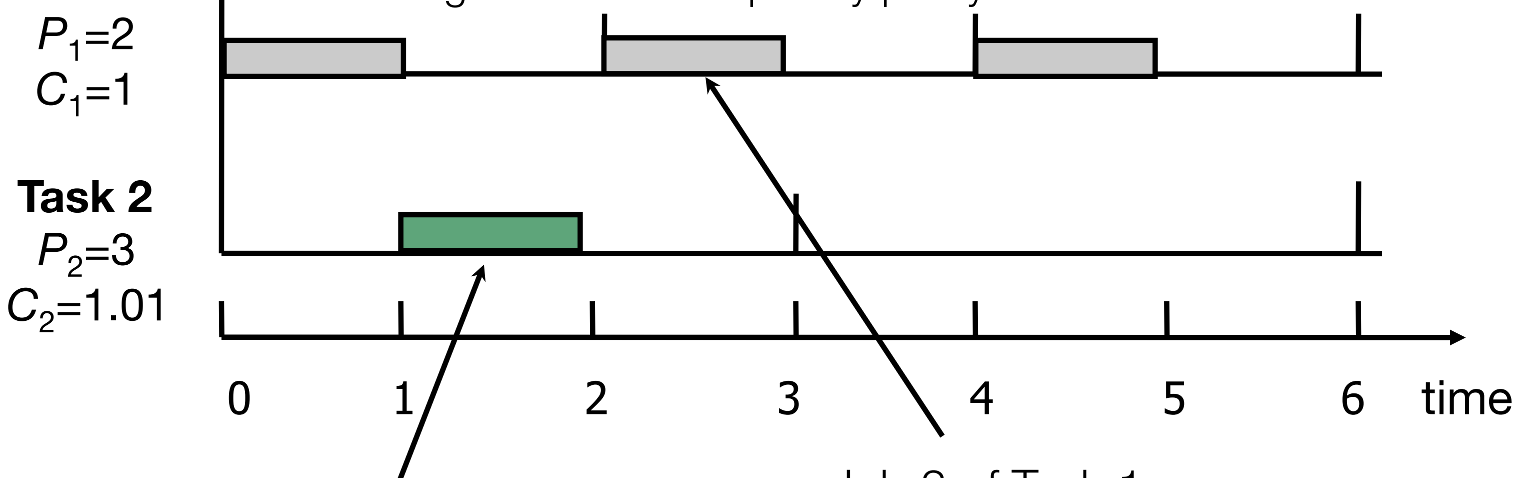
Job 1 of Task 1
Release time = 0
Relative deadline = 2
Absolute deadline = 0+2 = 2

Job 2 of Task 1
Release time = 2
Relative deadline = 2
Absolute deadline = 2+2 = 4

The release time of the first job of a task is also known as the **phase** of the task. The phase of Task 1 is 0.

A quick refresher

Job 2 of Task 1 preempts Job 1 of Task 2 at time=2 because we are following the rate monotonic priority assignment: a static priority policy.



Job 1 of Task 2
Release time = 0
Start time = 1
Relative deadline = 3
Absolute deadline = 0+3 = 3

Job 2 of Task 1
Release time = 2
Relative deadline = 2
Absolute deadline = 2+2 = 4

Schedulability analysis of periodic tasks

- Main problem
  - Given a set of periodic tasks, can they meet their deadlines?
  - Depends on scheduling policy
- Solution approaches
  - Utilization bounds (simplest)
  - Exact analysis (NP-Hard)
  - Heuristics
- Two most important scheduling policies
  - Earliest deadline first (dynamic)
  - Rate monotonic (static)

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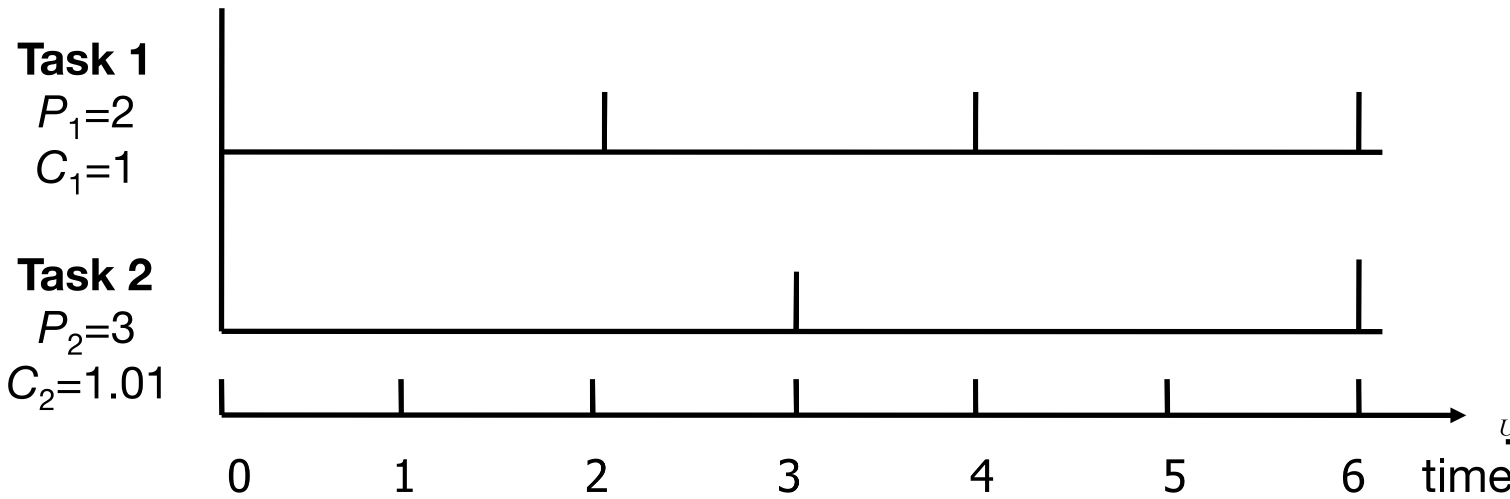
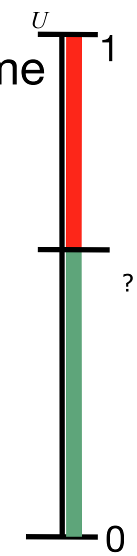
- Two most important scheduling policies
  - Earliest deadline first (Dynamic)
  - Rate monotonic (static)

Utilization bounds

- Intuitively,
  - The lower the processor utilization, U, the easier it is to meet deadlines.
  - The higher the processor utilization, U, the more difficult it is to meet deadlines.
- Question: Is there a threshold U_{bound} such that
  - When $U < U_{bound}$ deadlines are met
  - When $U > U_{bound}$ deadlines are missed

Example (Rate monotonic scheduling)





$$U = \frac{C_1}{P_1} + \frac{C_2}{P_2} = \frac{1}{2} + \frac{1.01}{3} \approx 0.833$$

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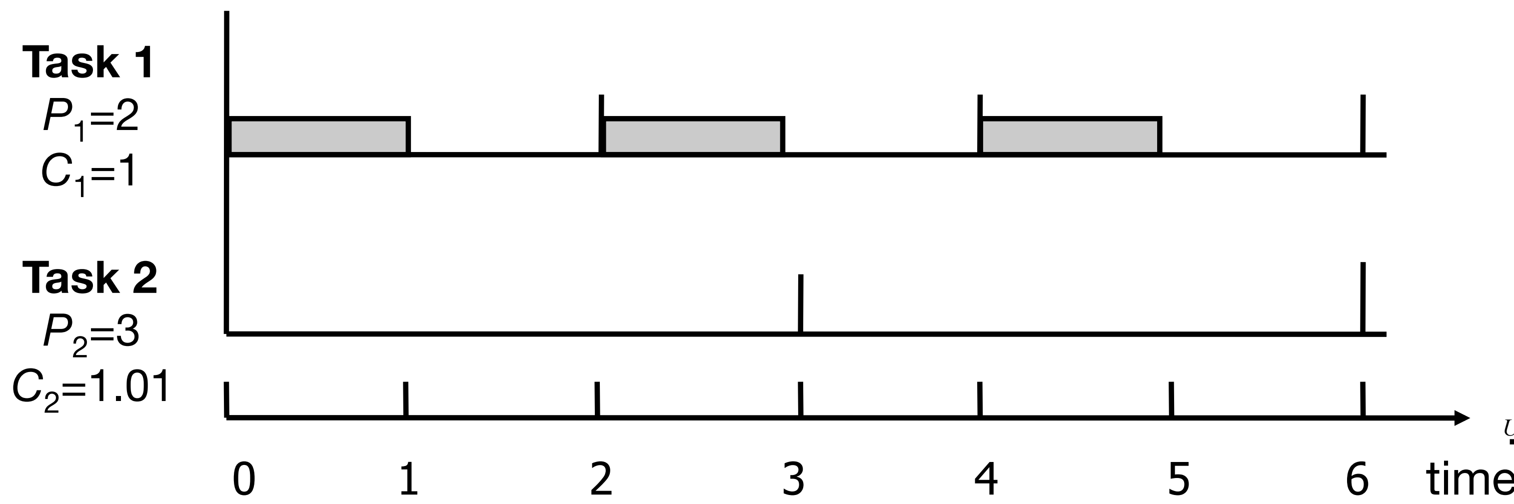

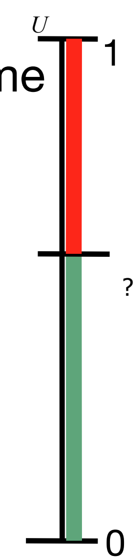

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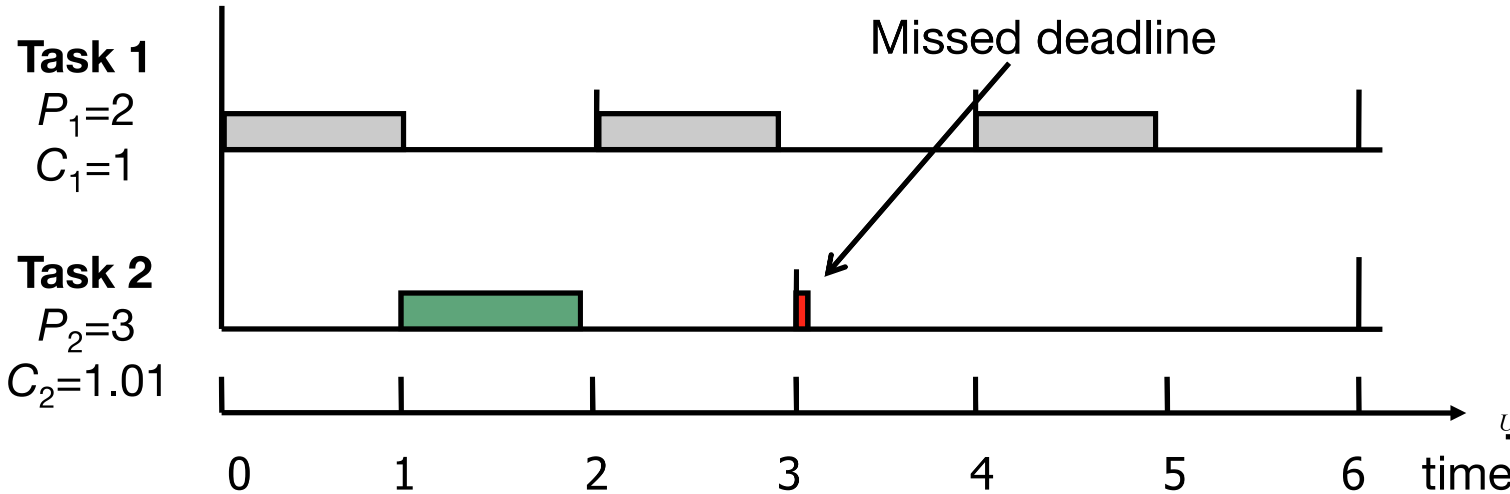

Unschedulable

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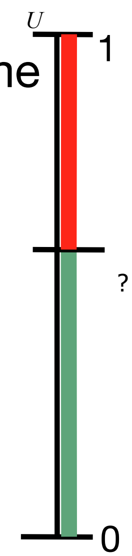
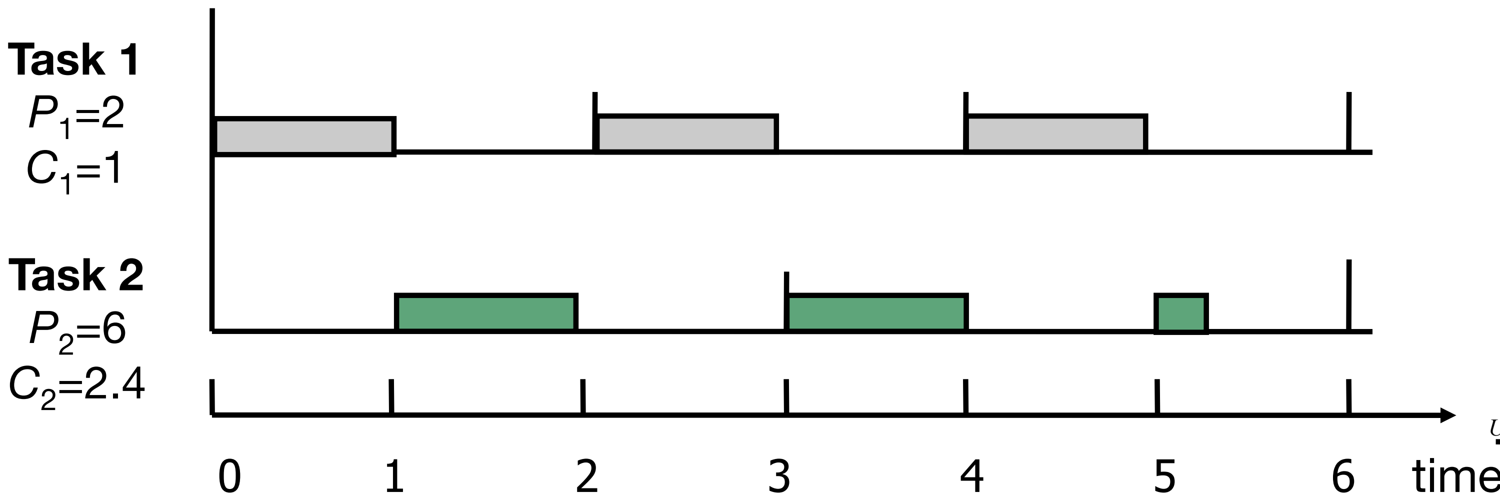
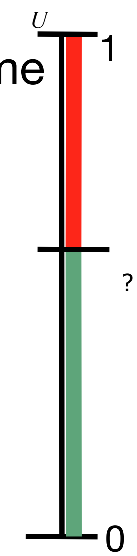


Another example (Rate monotonic scheduling)



$$U = \frac{C_1}{P_1} + \frac{C_2}{P_2} = \frac{1}{2} + \frac{2.4}{6} = 0.9$$

Schedulable



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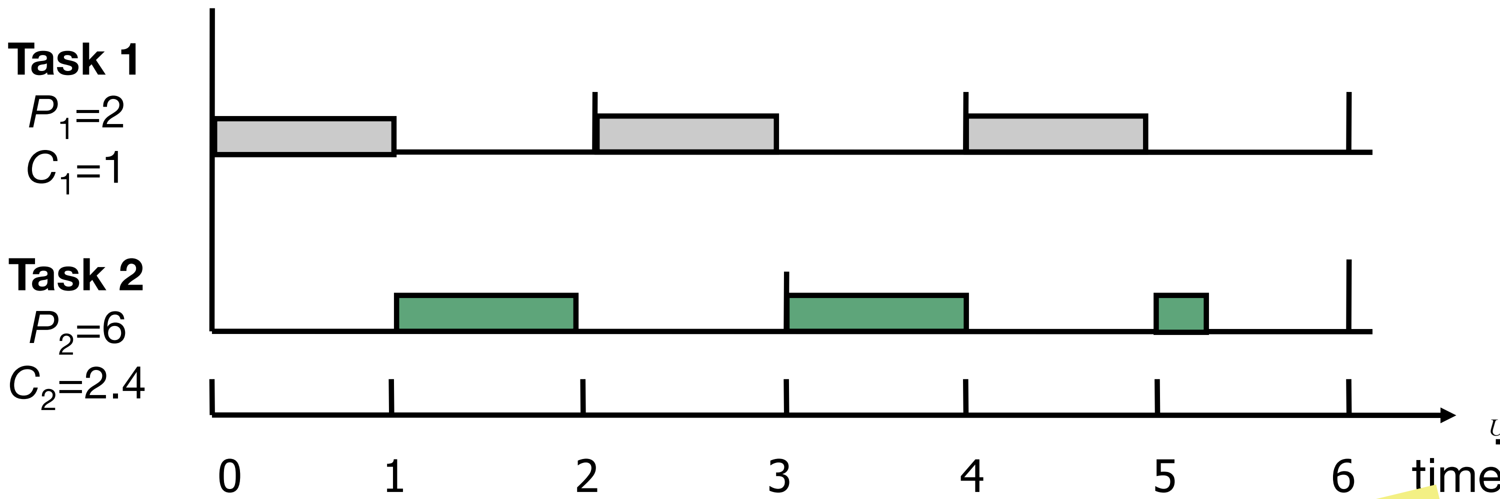

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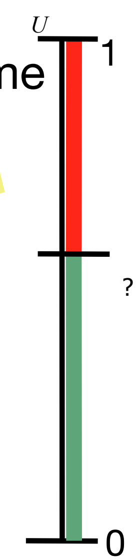

Schedulability depends on the task set.
No clean relationship between utilization and schedulability.

- **Question: Is th... U_{bound} such that**
  - **When U < U_{bound} deadlines are met**
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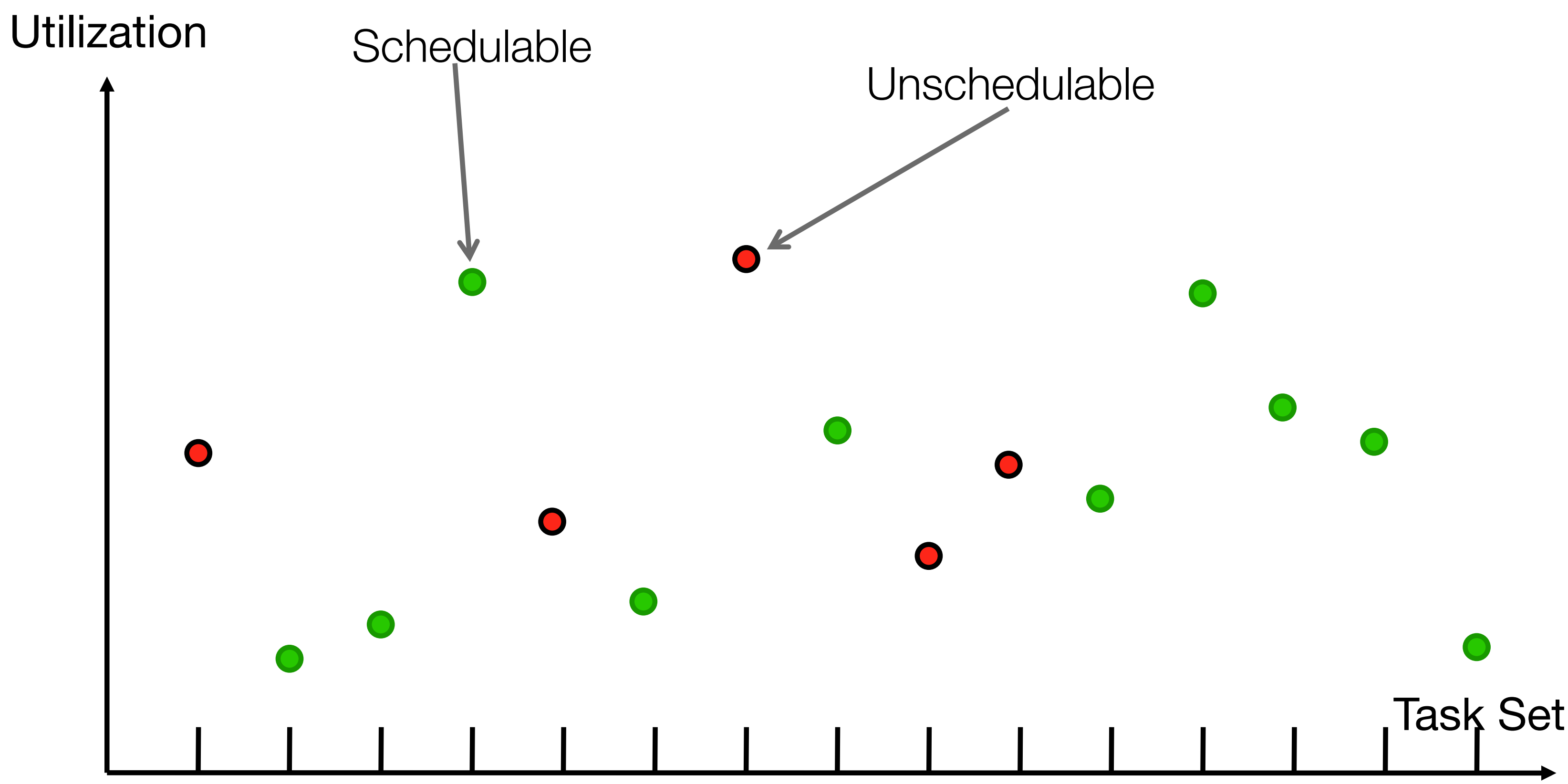
Visualizing schedulability



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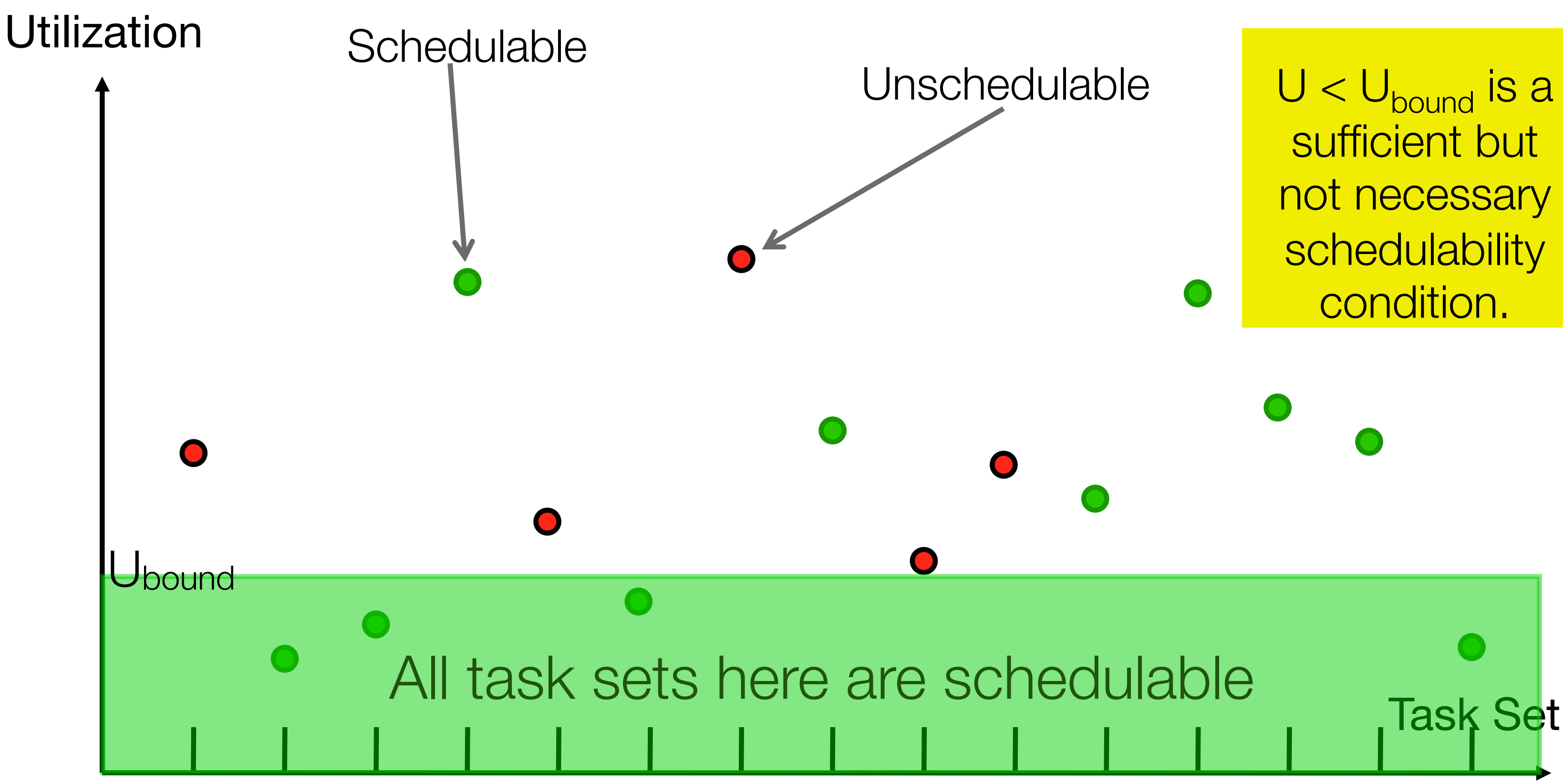
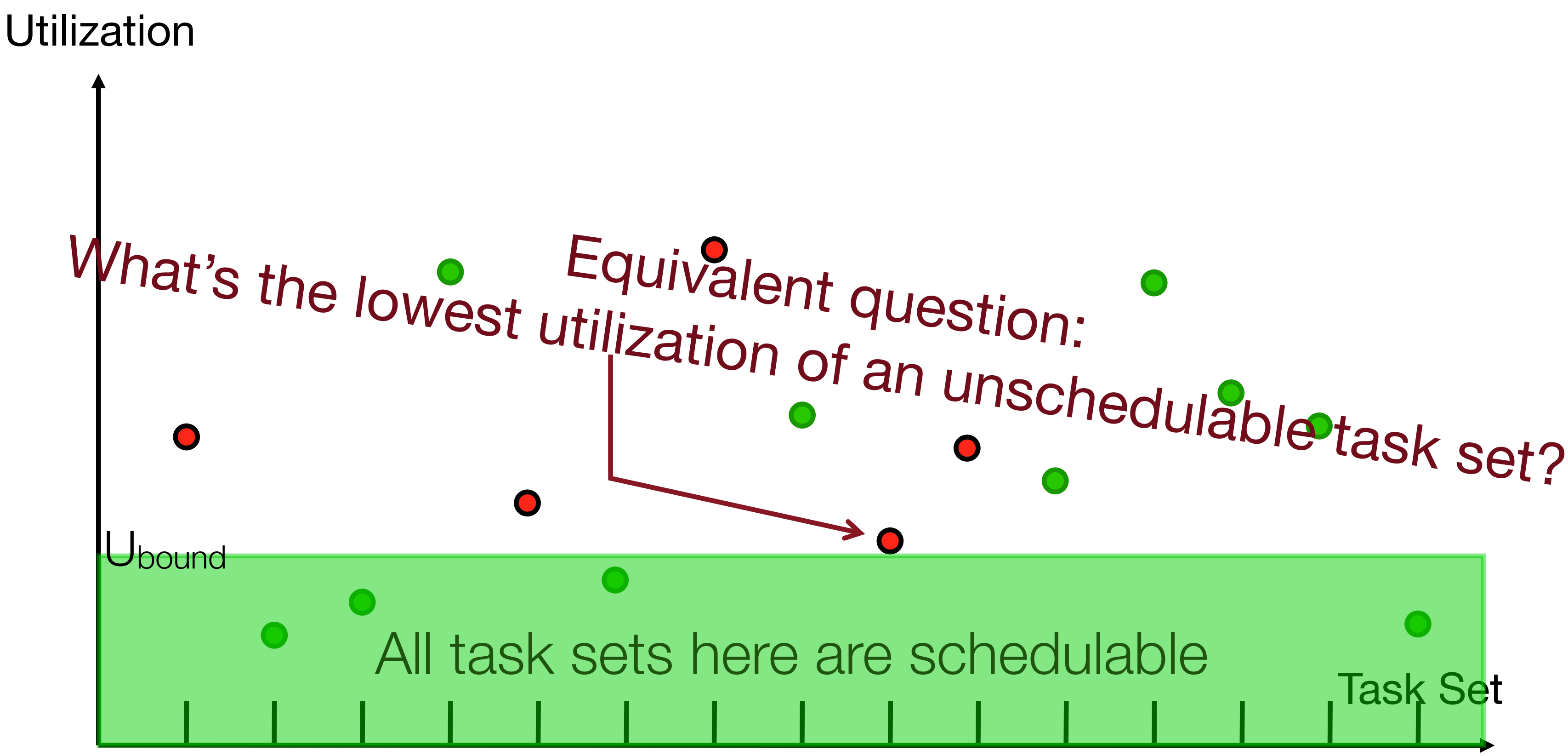


$U < U_{bound}$ is a sufficient but not necessary schedulability condition.

- Modified question: Is there a threshold U_{bound} such that
- When $U < U_{bound}$ deadlines are met
- When $U > U_{bound}$ deadlines may or may not be missed

Visualizing schedulability



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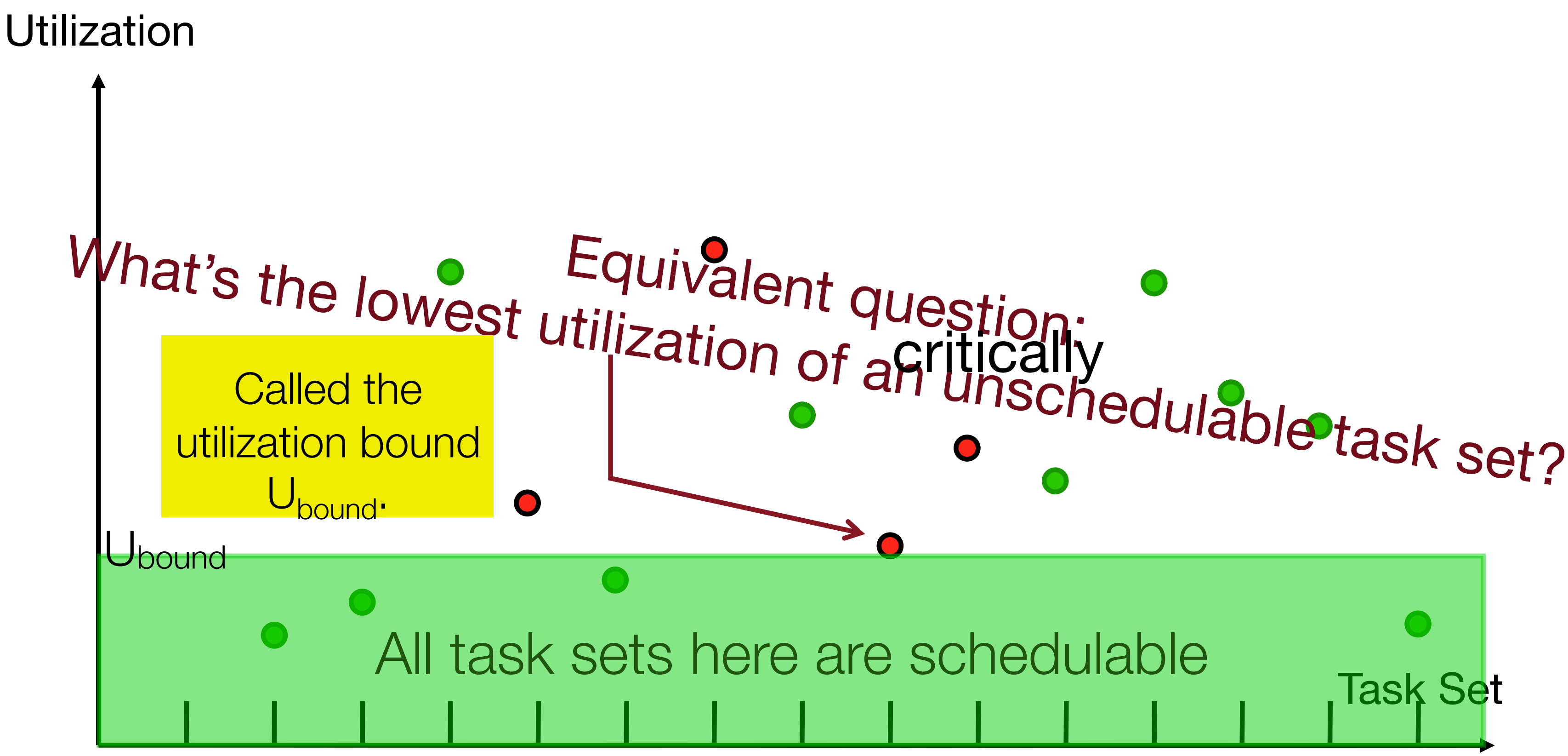

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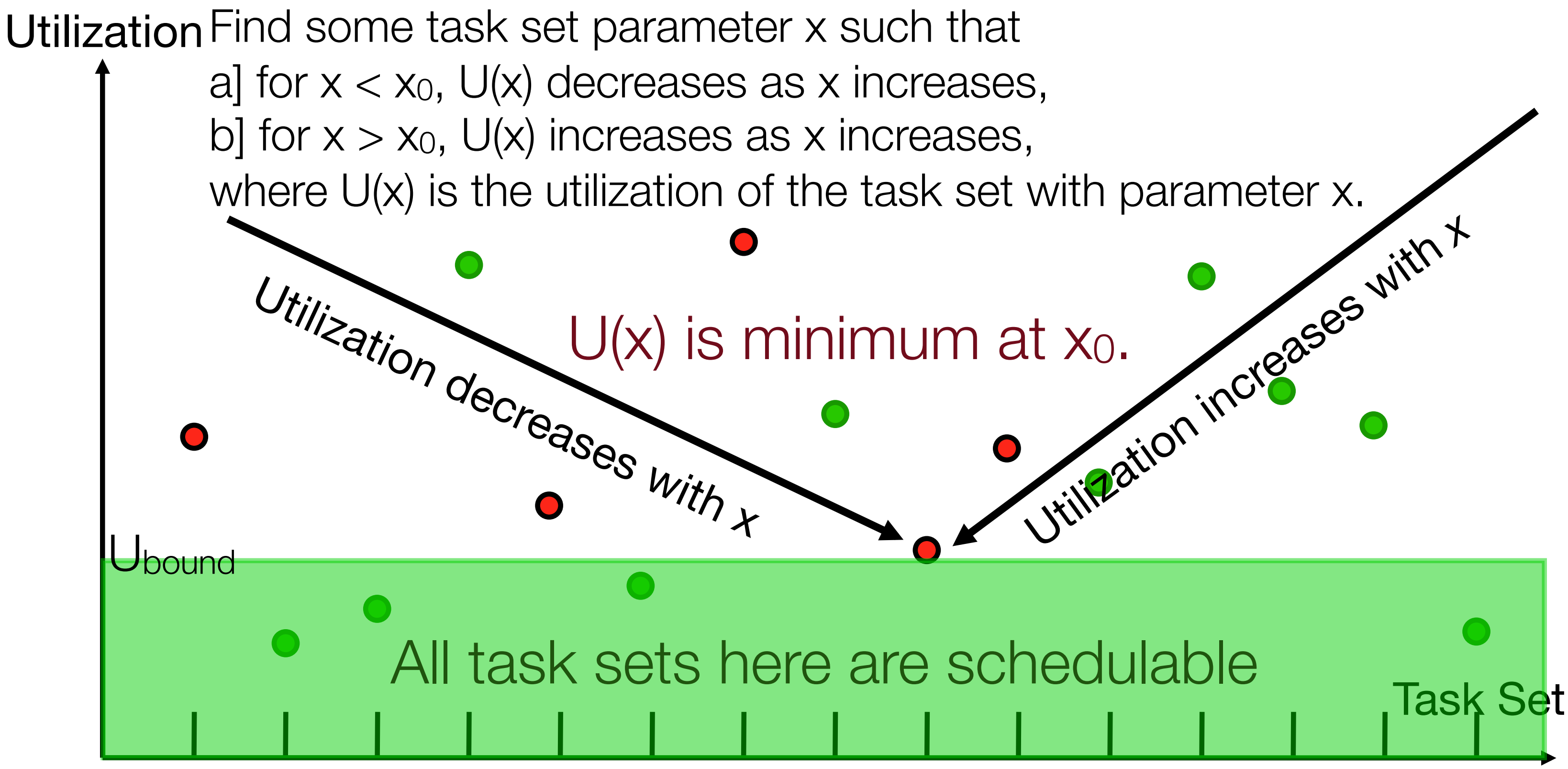
Finding the utilization bound for RM scheduling

Find some task set parameter x such that
a] for $x < x_0$, $U(x)$ decreases as x increases,
b] for $x > x_0$, $U(x)$ increases as x increases,
where $U(x)$ is the utilization of the task set with parameter x.



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Finding the utilization bound for RM scheduling

- **Consider the simplest case with two tasks**

Find some task set parameter x
such that
  Case (a): $x<x_o$ such that $U(x)$ decreases as x increases
  Case (b): $x>x_o$ such that $U(x)$ increases as x increases
Thus $U(x)$ is minimum when $x=x_o$
Find $U(x_o)$

Finding the utilization bound for RM scheduling

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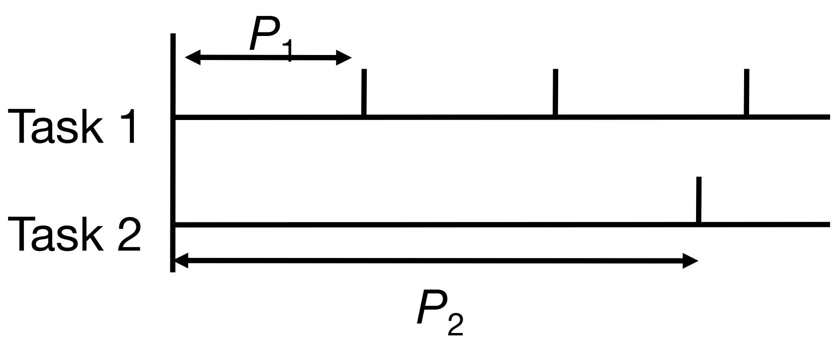

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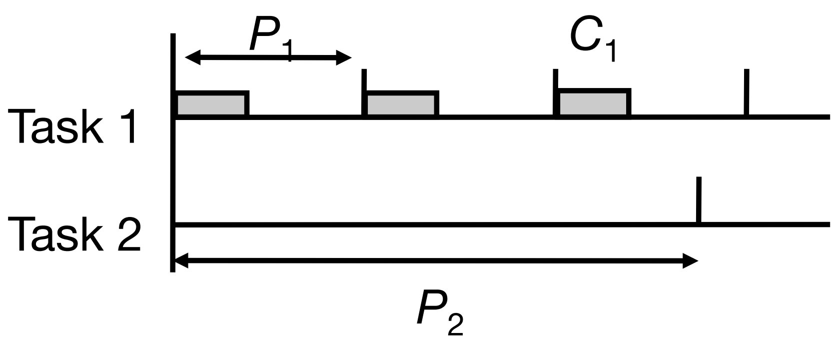

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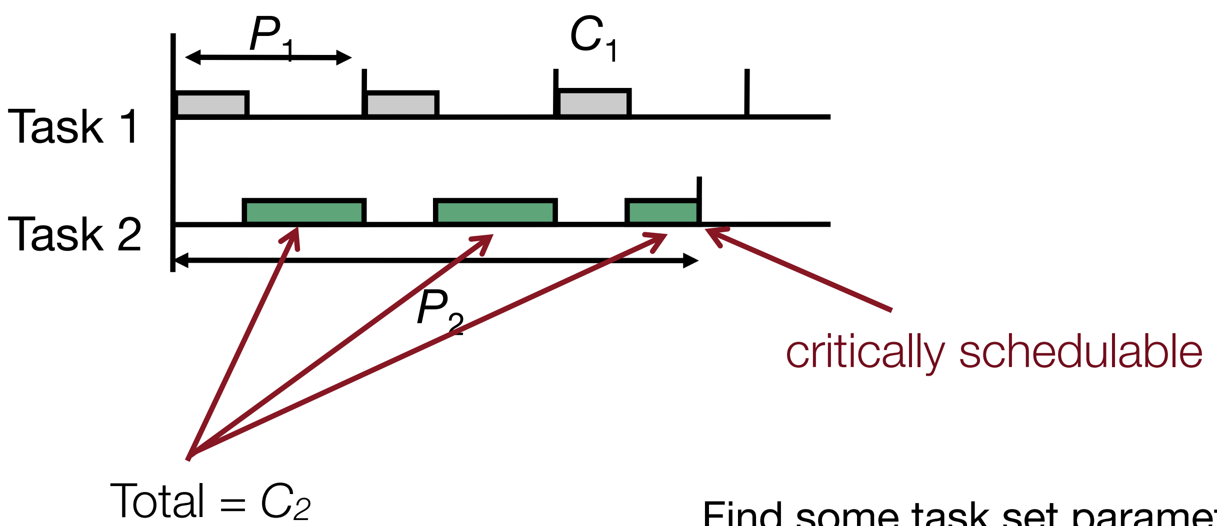

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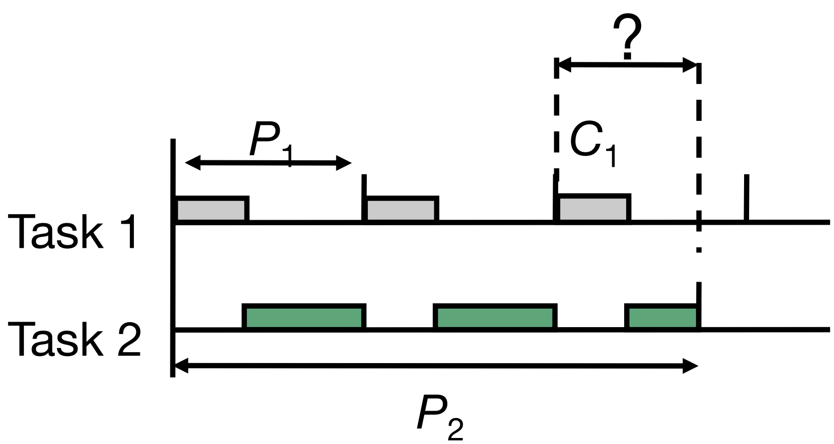

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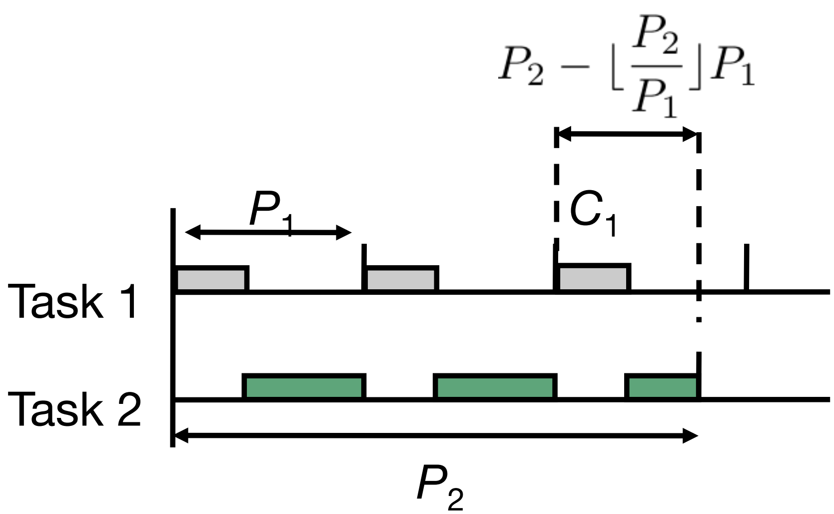

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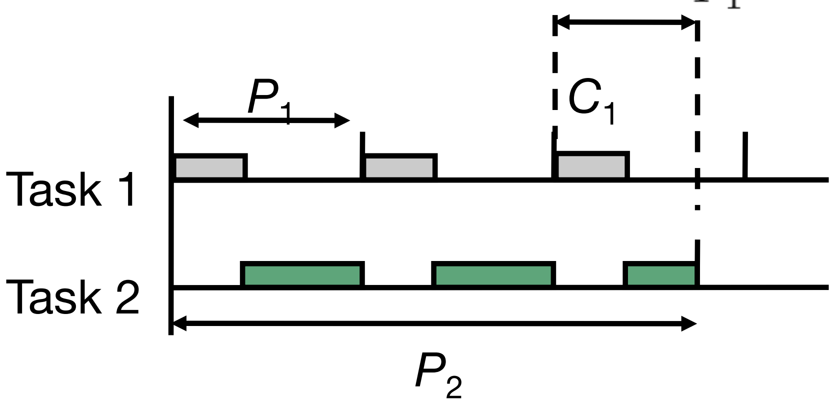
Finding the utilization bound for RM scheduling

- **Consider the simplest case with two tasks**
  - **There are two sub-cases…**

$$C_1 \leq P_2 - \lfloor \frac{P_2}{P_1} \rfloor P_1$$



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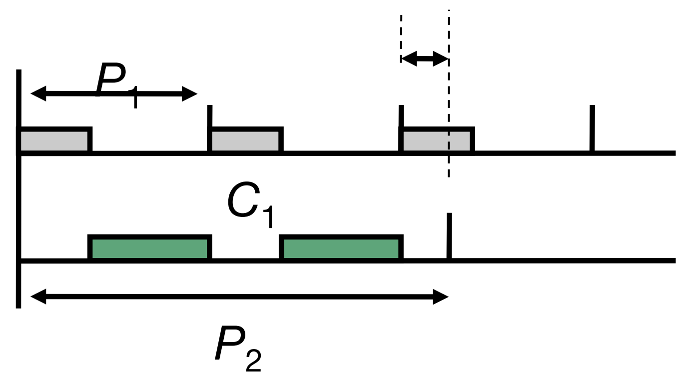

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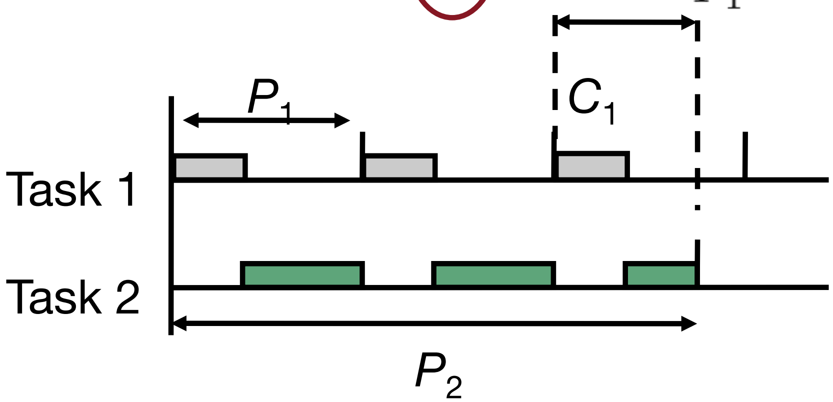

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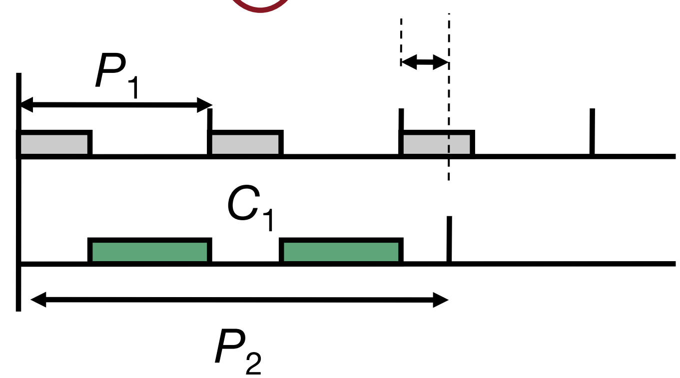

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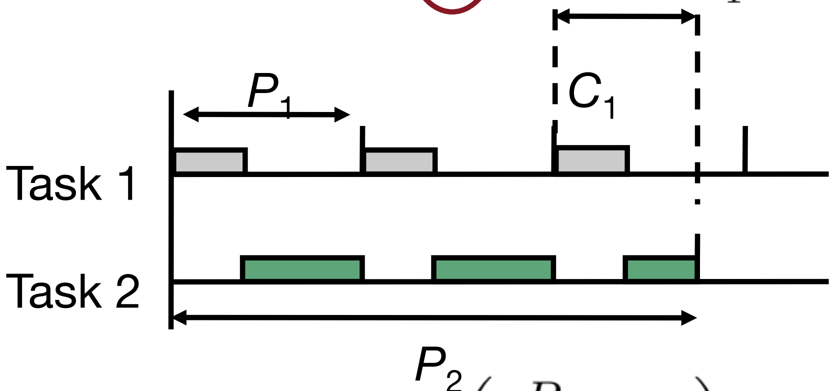
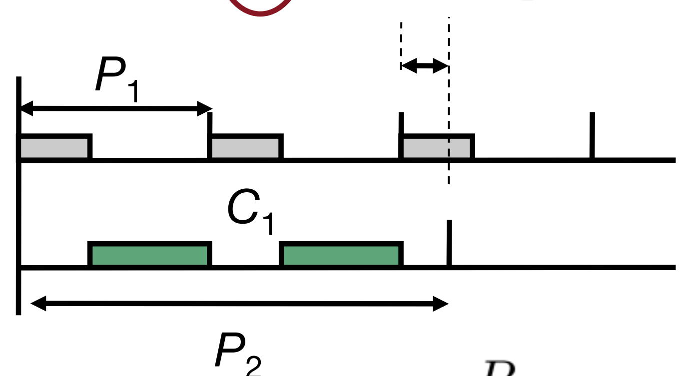


$$C_2 = P_2 - C_1\left(\lfloor \frac{P_2}{P_1} \rfloor + 1\right)$$

$$U = \frac{C_1}{P_1} + \frac{C_2}{P_2} = 1 + \frac{C_1}{P_2}\left[\frac{P_2}{P_1} - \lfloor \frac{P_2}{P_1} \rfloor - 1\right]$$

$$C_1 > P_2 - \lfloor \frac{P_2}{P_1} \rfloor P_1$$



$$C_2 = (P_1 - C_1)\lfloor \frac{P_2}{P_1} \rfloor$$

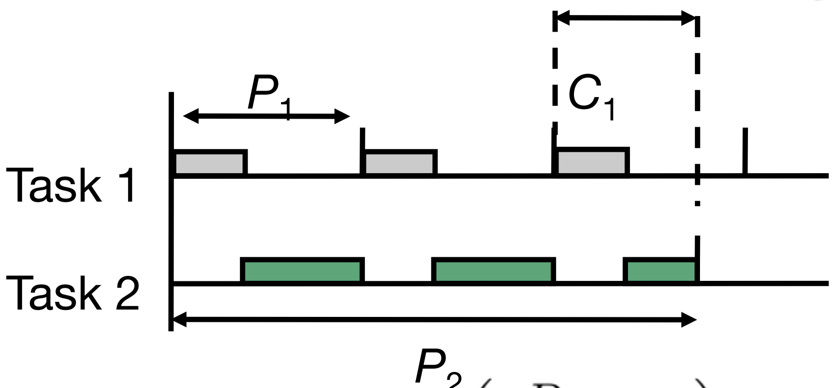
$$U = \frac{P_1}{P_2}\lfloor \frac{P_2}{P_1} \rfloor + \frac{C_1}{P_2}\left[\frac{P_2}{P_1} - \lfloor \frac{P_2}{P_1} \rfloor\right]$$

Finding the utilization bound for RM scheduling

The minimum utilization case

$$C_1 = P_2 - \lfloor \frac{P_2}{P_1} \rfloor P_1 = P_1 \left(\frac{P_2}{P_1} - \lfloor \frac{P_2}{P_1} \rfloor \right)$$

$$U = 1 + \frac{P_1}{P_2} \left(\frac{P_2}{P_1} - \lfloor \frac{P_2}{P_1} \rfloor \right) \left[\frac{P_2}{P_1} - \lfloor \frac{P_2}{P_1} \rfloor - 1 \right]$$



$$C_2 = P_2 - C_1 \left(\lfloor \frac{P_2}{P_1} \rfloor + 1 \right)$$

$$U = \frac{C_1}{P_1} + \frac{C_2}{P_2} = 1 + \frac{C_1}{P_2} \left[\frac{P_2}{P_1} - \lfloor \frac{P_2}{P_1} \rfloor - 1 \right]$$

To minimize U, we must have

$$\lfloor \frac{P_2}{P_1} \rfloor = 1$$

Why?

Finding the utilization bound for RM scheduling

The minimum utilization case

$$U = 1 + \frac{P_1}{P_2}\left(\frac{P_2}{P_1} - \lfloor\frac{P_2}{P_1}\rfloor\right)\left[\frac{P_2}{P_1} - \lfloor\frac{P_2}{P_1}\rfloor - 1\right]$$

To minimize U, we must have $\lfloor\frac{P_2}{P_1}\rfloor = 1$

$$U = 1 + \frac{(P_2/P_1 - 1)(P_2/P_1 - 2)}{P_2/P_1}$$

Then $\frac{dU}{d(P_2/P_1)} = 0 \Rightarrow \frac{P_2}{P_1} = \sqrt{2}$

Finally, U = 0.83

Note that $C_1 = P_2 - P_1 \text{ and } C_2 = 2P_1 - P_2$

Generalization for n tasks

$$\left.\begin{array}{rcl} C_1 & = & P_2 - P_1 \\ C_2 & = & P_3 - P_2 \\ C_3 & = & P_4 - P_3 \\ & \cdots & \end{array}\right\} U = \frac{C_1}{P_1} + \frac{C_2}{P_2} + \frac{C_3}{P_3} + \ldots$$

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$$\frac{dU}{d(P_2/P_1)} = 0; \frac{dU}{d(P_3/P_2)} = 0; \frac{dU}{d(P_4/P_3)} = 0; \ldots$$

Generalization for n tasks

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$$\frac{dU}{d(P_2/P_1)} = 0; \frac{dU}{d(P_3/P_2)} = 0; \frac{dU}{d(P_4/P_3)} = 0; \ldots$$

We can then obtain

Liu & Layland, 1973

$$\frac{P_{i+1}}{P_i} = 2^{1/n} \Rightarrow U = n(2^{1/n} - 1)$$

For large n: $\lim_{n\to\infty} U = \lim_{n\to\infty} n(2^{1/n} - 1) = \ln 2 \approx 0.69$

Lecture summary

- Understanding utilization bounds
- The utilization bound for rate-monotonic scheduling
- For RM scheduling the bound decreases with the number of tasks, approaching an asymptotic limit of 0.69
- Coming up: Why is RM priority assignment the optimal static priority policy? Are there better schedulability tests?