ASSIGNMENT 4

DUE DATE: NOV 15, 2011

i) Show that if F is a finite field, the Milnor K-groups $K_i^M(F)$ are trivial for $i \geq 2$.

ii) Show that for any profinite group G and G-module M, the Galois cohomology groups $H_c^i(G, M)$ are torsion.

iii) Let D be a division algebra, finite dimensional over its centre F, and let E be any finite extension of F which is a splitting field of D, i.e. $E \otimes_F D \simeq M_n(E)$. For any splitting field E, the inclusion of D in $M_n(E)$ and $M_r(D)$ in $M_nr(E)$ induces maps

$$D^\times \subset GL_n(E) \xrightarrow{\det} E^\times$$

and

$$GL_r(D) \to GL_{nr}(E) \xrightarrow{\det} E^\times$$

whose image lies in the subgroup $F^\times$ of $E^\times$. These induced maps $D^\times \to F^\times$ and $GL_r(D) \to F^\times$ are called the reduced norms Nrd for D.
a) Show that Nrd is independent of the splitting field E.
b) Show that the norm (or transfer) map $K_1(D) \to K_1(F)$ is n times the reduced norm Nrd where $d = n^2$ is the dimension of D over its centre F.

iv) Let F be a field with a discrete valuation v and let k_v be the residue field of the valuation. Consider the map

$$F^\times \times F^\times \xrightarrow{\delta_v} k_v^\times,$$

defined by

$$\delta_v(r, s) = (-1)^{v(r)v(s)} \overline{\left(\frac{s^{v(r)}}{r^{v(s)}}\right)}.$$

Show that this defines a 'tame symbol' homomorphism from $K_2(F) \to K_1(k(v))$.