ASSIGNMENT 6

DUE DATE: NOV 22, 2011

1) Show that a group of order 45 is abelian. Determine the number of isomorphism classes of groups of order 45. 10pts

2) Let G be a group of order 80. Show that in G either a 5-Sylow subgroup is normal or a 2-Sylow subgroup is normal. 10pts

(**Hint**: First compute the possible number of 5-Sylow and 2-Sylow subgroups. Study the possible intersections of the Sylow subgroups and show that if there is more than one 5-Sylow subgroup and 2-Sylow subgroup, taking the unions of all of them and counting the distinct elements in this union gives more elements than the cardinality of G which is a contradiction.)

3) Show that if G is a group of order 385, prove that its 7-Sylow subgroup is contained in the centre. Write down the possible p-Sylow subgroups and the possible numbers of p-Sylow subgroups. 10pts

(**Hint**: If P_7 is a 7-Sylow subgroup, then look at the action of G on P_7 by conjugation and use that the index of the centralizer $C_G(P_7)$ should be a subgroup of $\mathrm{Aut}(P_7)$ and examine when this is possible by looking at their cardinalities).

4) Show that if X is a topological space, the set $C(X, \mathbb{R})$ whose elements are continuous functions from X to the set $\mathbb{R}$ of real numbers has a ring structure. Fix a point x in X. Check that the set

$$I = \{f \in C(X, \mathbb{R}) \text{ such that } f(x) = 0$$

is an ideal. 10pts

5) Show that if n is a composite divisor, then the ring $\mathbb{Z}/n$ has zero divisors. Write down the zero divisors in $\mathbb{Z}/16,\ \mathbb{Z}/14$. 10pts

6) Show that the set $\{\mathbb{Z}[\sqrt{-2}] = (a + b\sqrt{-2}) \mid a,\ b \in \mathbb{Z}\}$ has a ring structure. 10pts