1 Oct 20

UBC Math 312 – Quiz 1 Answer Key

) The greatest common divisor of two integers a & b (that are not both zero) is the largest integer which divides both a & b.

$$70 = 2 \times 5 \times \boxed{7}$$
$$98 = 2 \times \boxed{7} \times 7$$
$$105 = 3 \times 5 \times \boxed{7}$$

Thus, gcd(70, 98, 105) = 7.

2) Let $\pi(x)$ denote the number of primes not exceeding x where $x \in \mathbb{R}^{+}$.

The Prime Number Theorem states that as $x \longrightarrow \infty$,

$\pi(x) \longrightarrow \dfrac{x}{\log x}$, where $\log x$ denotes the natural logarithm of x.

The asymptotic formula for the n^{th} prime is $(n)(\log n)$, where $\log n$ denotes the natural logarithm of n.

3) <u>Base Case: $n = 4$.</u>

$$4^2 = 16 < 24 = 4!$$

So the statement is true for $n = 4$.

<u>Induction Step</u>

Assume $k^2 < k!$ for some $k \in \mathbb{N}$, $k \geq 4$.

Then $(k+1)^2 = k^2 + 2k + 1$

$< k! + 2k + 1$ {by assumption}

$< k! + (k-1)k! + k!$ { since for $k \geq 4$ we have
$2 < k-1$
$k < k!$ and
$1 < k!$ }

$= k!\left[1 + (k-1) + 1\right]$

$= (k+1)!$

<u>Conclusion</u>

By mathematical induction, $n^2 < n!$ for all $n \geq 4$, $n \in \mathbb{N}$.

4) i) <u>False</u>.

Consider $a = 2$, $r = 3$. $(2,3) = 1$. But then the set $\{2, 2(3), 2(3)^2, 2(3)^3, \ldots\}$ contains only even numbers, so the only prime in the set is 2.

[Another cool counterexample is:
Let $a = p$, prime, and $r = 1$. Then $(p,1) = 1$.
What happens then?]

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ii) <u>True</u>. (in fact, true for any $n \in \mathbb{N}$)
This is a theorem proven in the textbook.
See the introductory section on Prime Numbers.

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iii) <u>True</u>.
This was proven in the textbook.
See the section on Greatest Common Divisors.

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iv) <u>False</u>.

This was proven in the textbook.
See the section on The Fundamental Theorem of Arithmetic.