$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 2 & 1 & -1 & 0 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 10 \\ 12 \end{pmatrix} = \begin{pmatrix} x_4^* \\ x_5^* \end{pmatrix}$$

$$c = (2, 3, 1, 0, 0). \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Revised simplex.

step 1: Solve $yB = c_B$

$$(y_1\ y_2)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = (0,0) \Rightarrow y = (0,0).$$

step 2: $z_1 = c_1 - ya_1 = 2 - (0,0)\begin{pmatrix} 1 \\ 2 \end{pmatrix} = 2$

$z_2 = c_2 - ya_2 = 3 - (0,0)\begin{pmatrix} 1 \\ 1 \end{pmatrix} = 3$

$z_3 = 1 - y \cdot a_3 = 1 - (0,0)\begin{pmatrix} 1 \\ -1 \end{pmatrix} = 1$

The entering variable is x_2.

Step 3: Solve $Bd = a_2$.

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow d = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

step 4: Find maximal t such that

$$x_B^* - td = \begin{pmatrix} 10 - t \\ 12 - t \end{pmatrix} \geq 0 \quad \Rightarrow t = 10, \text{ leaving variable} = x_4.$$

step 5: $\begin{pmatrix} x_2^* \\ x_5^* \end{pmatrix} = \begin{pmatrix} 10 \\ 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}.$

Second iteration:

step 1: Solve $yB = c_B$.

$$(y_1\ y_2)\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = (3, 0) \Rightarrow y = (3, 0)$$

step 2: $z_1 = 2 - (3,0)\begin{pmatrix} 1 \\ 2 \end{pmatrix} = -1$

$$z_3 = 1 - (3,0)\begin{pmatrix} 1 \\ -1 \end{pmatrix} = -2$$

$$z_4 = 0 - (3,0)\begin{pmatrix} 1 \\ 0 \end{pmatrix} = -3$$

So the solution $\begin{pmatrix} x_2^* \\ x_5^* \end{pmatrix} = \begin{pmatrix} 10 \\ 2 \end{pmatrix}$, that is

$x_1 = 0,\ x_2 = 10,\ x_3 = 0,\ x_4 = 0,\ x_5 = 2$ is optimal.

The optimal value for the objective function

is $c \cdot x_0^* = 2 \cdot 0 + 3 \cdot 10 + 1 \cdot 0 + 0 \cdot 0 + 0 \cdot 2 = 30$.