Math 110 Practice Exam #2

	002	
name (printed)	*section*	*student number*

I have read and understood the instructions below:

signature

Instructions:

1. Calculators are not permitted.
2. There are 11 pages (including this cover page) in the test. **Justify every answer, and clearly show your work.** Unsupported answers will receive no credit.
3. You will be given **90 min** to write this test. Read over the test before you begin.
4. **Academic dishonesty:** Exposing your paper to another student, copying material from another student, or representing your work as that of another student constitutes academic dishonesty. Cases of academic dishonesty may lead to a zero grade in the test, a zero grade in the course, and other measures, such as suspension from this university.

Question	value	score
1	10	
2	16	
3	6	
4	9	
5	6	
6	6	
7	4 bonus	
Total	**53**	

Question 1:[10 points] State whether each of the following statements is true. If it is, explain why. If it is not, explain why not (for example, by providing a counterexample).

(a)[**2 marks**] A function can have zero, one, or two horizontal asymptotes.

(b)[**2 marks**] A polynomial $p(x)$ must have an absolute minimum value on the interval [0, 1].

(c)[**2 marks**] If a function f is continuous on an interval $[a, b]$, then there must be a point c in $[a, b]$ so that the slope of the tangent line to $y = f(x)$ at c is equal to the slope of the secant line (the line joining $(a, f(a)$ and $(b, f(b))$.)

(d)[**2 marks**] The graph of a continuous function can never cross over an horizontal asymptote.

(e)[**2 marks**] If f is a continuous function and $f'(x) = 0$ for all values of x, then $f(x) = 0$.

Question 2: [16 points] Let $f(x) = e^{2/x}$. Consider the curve $y = f(x)$.

(a)[2 points] Find the x-intercepts and y-intercepts of the curve, if there are any.

(b)[2 points] Find the horizontal and vertical asymptotes.

(c)[4 points] Determine where f is increasing and where it is decreasing, and find its local minima and maxima, if these exist.

(d)[**4 points**] Determine where the curve is concave up and where it is concave down, and find its inflection points, if these exist.

(e)[**4 points**] Sketch the curve, showing the properties determined in parts (a) through (d).

Question 3: [6 points] The doubling time for a population of the Vibrio Cholerae bacterium (the micro-organism responsible for cholera) can be as short as 20 minutes.

If we start with 1/2 gram of these bacteria, what is the maximum mass of bacteria which could be present after 3 hours? After 8 hours?

Question 4: [9 points] The length l of a rectangle is decreasing at a rate of 2cm/sec, while its width w is increasing at a rate of 2cm/sec. At some moment, $l = 12$cm and $w = 5$cm.

(a)[**3 points**] Find the rate of change of the area A with respect to time. Be sure to indicate whether the quantity is increasing or decreasing.

(b)[**3 points**] Find the rate of change of the perimeter P with respect to time. Be sure to indicate whether the quantity is increasing or decreasing.

(c)[**3 points**] Find the rate of change of the length D of the diagonal with respect to time. Be sure to indicate whether the quantity is increasing or decreasing.

Question 5: [6 points]

(a)[2 points] State the Mean Value Theorem, including all hypotheses, and giving a precise conclusion.

(b)[2 points] Suppose that f is differentiable on the interval (a, b), and that $f'(x) = 0$ for all x in (a, b). Take a smaller interval $[x_0, x_1]$ contained in (a, b), where $x_0 < x_1$. What does the Mean Value Theorem tell you about $f(x_0)$ and $f(x_1)$?

(c)[2 points] Note that the points x_0 and x_1 in part (b) are arbitrarily chosen. Use this observation to prove that f is a constant function.

Question 6: [6 points] A baseball thrown into the air at an angle θ, with initial speed v travels a distance R over level ground according to the following formula:

$$R = \frac{v^2}{g} \sin 2\theta,$$

where g is a constant factor.

Find the angle θ which gives the maximizes the distance that the ball can be thrown.

Question 7: [4 bonus points] The Mean Value Theorem lets us prove that if f and g are continuous and $f'(x) = g'(x)$ for each point x in an open interval (a, b), then there is a constant C so that $f(x) = g(x) + C$.

Use this fact to prove that $\ln bx = \ln b + \ln x$ (product rule for logarithms).

This page may be used for rough work. It will not be marked.