



Math 110 Practice Exam #1

name (printed)

section

student number

I have read and understood the instructions below:

signature

Instructions:

1. Calculators are not permitted.
2. There are 14 pages (including this cover page) in the test. **Justify every answer, and clearly show your work.** Unsupported answers will receive no credit.
3. You will be given **90 min** to write this test. Read over the test before you begin.
4. **Academic dishonesty:** Exposing your paper to another student, copying material from another student, or representing your work as that of another student constitutes academic dishonesty. Cases of academic dishonesty may lead to a zero grade in the test, a zero grade in the course, and other measures, such as suspension from this university.

Question	value	score
1	10	
2	4	
3	12	
4	12	
5	6	
6	5	
7	8	
8	4	
9	6	
10	4 bonus	
Total	67	

Question 1:[10 points] State whether each of the following statements is true. If it is, explain why. If it is not, explain why not (for example, by providing a counterexample).

(a)[2 marks]

$$\frac{d}{dx} \sec^2 x = \tan^2 x$$

(b)[2 marks] If $f(y)$ and $g(x)$ are differentiable,

$$\frac{d}{dx}(f \circ g)(x) = \frac{df}{dy} \circ \frac{dg}{dx}$$

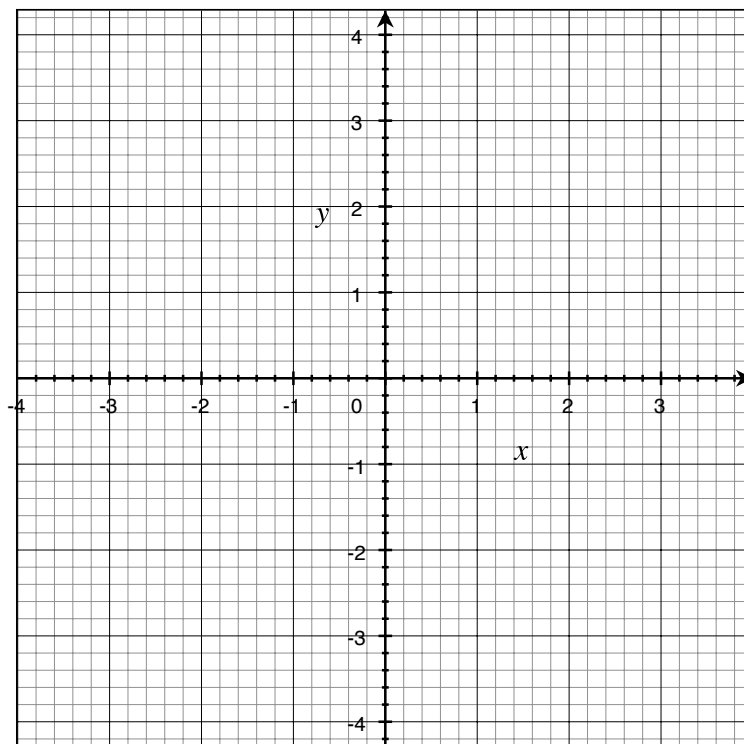
(c)[2 marks] If f is continuous at a then so is $|f|$.

(d)[2 marks] If $f'(c) = 0$ and $f''(c) > 0$, then f has a local maximum at c .

(e)[2 marks] The derivative of a continuous function is continuous.

Question 2: [4 points] Let $f(x) = x + 1$, let $g(x) = x^2$, and let $h(x) = \frac{1}{x}$.

(a)[2 points] Sketch the graph of $g \circ f$.



(b)[2 points] Is $h \circ g$ an even function? Explain your reasoning.

Question 3: [12 points] Evaluate the following limits.

(a)[3 points] $\lim_{x \rightarrow 2} \frac{x - 1}{x^2 - 1}$

(b)[3 points] $\lim_{x \rightarrow \infty} \frac{6x^2 + 2x + 1}{6x^2 - 3x + 4}$

(c)[3 points] $\lim_{x \rightarrow \infty} \frac{\sin x}{1 + x^2}$

(d)[3 points] $\lim_{x \rightarrow 0} \frac{\sin x}{x^2}$

Question 4: [12 points] For each of the following curves, find y' . (You do not need to simplify your answer.)

(a)[3 points] $y = \frac{3x^2}{2 - x^3}$

(b)[3 points] $x^2 - 4xy - y^2 = 4$

(c)[3 points] $y = x^{\tan x}$

(d)[3 points] $y = \ln(\sin(\tan(x)))$

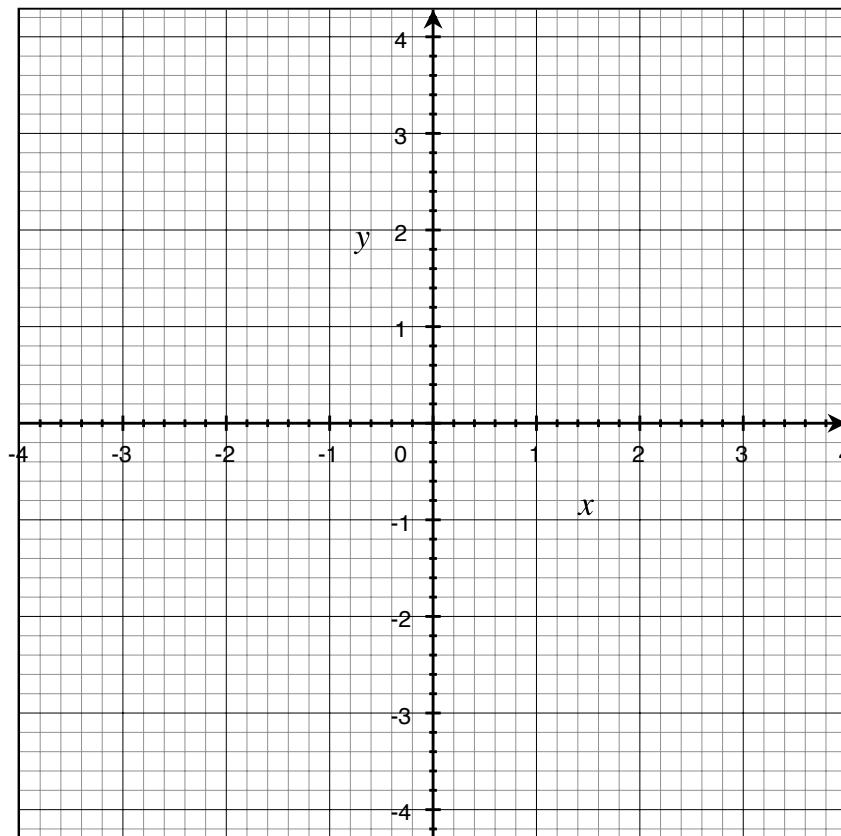
Question 5: [6 points] Let $f(x) = x^2 + 3x$.

(a)[4 points] Use any of the limit definitions of derivative to find $f'(-4)$.

(b)[2 points] Find an equation of the tangent line to $f(x)$ at $x = -4$.

Question 6: [5 points] Sketch a function f satisfying each of the following conditions:

- a) f has a vertical asymptote at $x = 1$.
- b) $f(x) < 0$ if $x > 1$, and $f(x) > 0$ if $x < 1$
- c) $\lim_{x \rightarrow -2^+} f(x) = 0$
- d) $\lim_{x \rightarrow -2^-} f(x) = 2$
- e) $f(-2) = 1$



Question 7: [8 points] Given $y = x^4 + 2x^3 - 3x^2 - 4x + 4$.

(a)[4 points] Find the intervals on which y is increasing and decreasing

(b)[4 points] Find the local and absolute maxima and minima of y , if these exist.

Question 8: [4 points] Prove that $x + 2 \cos x = 0$ has at least one solution. Mention any theorems you use.

Question 9: [6 points] The cable of a suspension bridge is attached to supporting pillars 250 metres apart. The cable hangs in the shape of a parabola, with the lowest point 50 metres below the points of suspension.

(a)[3 points] Write an function which models the parabola mentioned in the problem. Hint: draw a picture; use symmetry.

(a)[2 points] Find the slope of the cable at the point of attachment to the pillar.

(b)[1 points] Express the angle between the pillar and the cable at the point of attachment in terms of trigonometric functions. You need not compute a numerical value.

Question 10: [4 bonus points] Prove that the curves given by $5y - 2x + y^3 - x^2y = 0$ and $2y + 5x + x^4 - x^3y^2 = 0$ intersect at right angles at the origin.

This page may be used for rough work. It will not be marked.