The UNIVERSITY OF BRITISH COLUMBIA
Department of CHEMICAL and BIOLOGICAL ENGINEERING
CHBE 241: MATERIAL and ENERGY BALANCES
MIDTERM - EXAMINATION 16W
October 6, 2016; 12:35-13:50 PM

Name: $\qquad$

## Student ID:

$\qquad$

## Instructions:

The midterm is out of $\mathbf{1 0 0}$ points, use your time wisely

- Hand calculators (including programmable calculators) are allowed - The only allowed material is the three page formula sheet provided to you with this exam. You can detach the formula sheet for easier handling. Solve the $\mathbf{2}$ problems individually. - If any of these rules are not respected, it will be dealt with according to University Policy on student ethics during examination.

Please return this sheet with the exam in your exam booklet. PDF copies of the midterm questions will be available online following the exam.

1. Maple syrup is produced by concentrating the sap from maple trees. An initial stream of sap containing only sugar and water is fed into a reverse osmosis unit to remove pure water and concentrate the sugar. Following that the sugar solution is further concentrated in an evaporator which again removes pure water.
a) Draw a flowchart for this process showing the process units and labelling what components are contained in each stream. (Note that you do not need to include the values from parts $b$ and c) [15 points]
b) The initial stream of sap enters the process at a total flow rate of $75.0 \mathrm{~L} / \mathrm{min}$ and has a specific gravity of 1.035 , it contains 12000 ppm (by weight) of sugar, what is the flow rate of water into the process in $\mathrm{kg} / \mathrm{min}$ ? [10 points]
c) The entire process produces a final solution containing 67.0 weight percent sugar. Of all the water removed in the process, $70.0 \%$ is removed by the reverse osmosis unit and the other $30.0 \%$ removed by the evaporator. How much water is removed in the reverse osmosis unit in kg/min? [15 points]
2. Gypsum, a solid mineral, is used in the production of drywall in Vancouver. The gypsum entering the plant contains gypsum and $2.00 \mathrm{wt} \%$ water and is simultaneously crushed and dried in a hammer mill. The gypsum is dried in the hammer mill using a pure air stream containing no water. Two streams exit the hammer mill, one solid and one gas. The solid stream contains $0.50 \mathrm{wt} \%$ water and the remainder as gypsum. The gas stream contains all the air entering, the removed water and $1.00 \mathrm{wt} \%$ of gypsum, as fine particles.

In order to recover the gypsum in the air, and remove the fine particles, the gas stream exiting the hammer mill is sent to a cyclone unit. Two streams emerge from the cyclone, one stream of solids containing only gypsum, with 72.0 percent (by weight) of the gypsum particles entering, and the other stream of gas containing all the air and water entering and the remaining gypsum particles. The gas stream exiting the cyclone is sent for further processing elsewhere in the plant. The gypsum recovered in the solid stream exiting the cyclone is mixed in a mixer unit with the solids stream which left the hammer mill. The combination of these solids streams is then sent for further processing elsewhere in the plant.
a) Draw a process flowchart containing 3 units, labelling all streams with known and unknown variables. List the extra relationships between variables that are known. [You should not need to calculate any values] [20 points]
b) Assuming a basis of 50.0 metric tonnes a day of gypsum entering the plant, perform a degree of freedom analysis on each individual unit as well as the overall process. [You should not need to calculate any values] [20 points]
c) A pressure gauge reads the pressure of the gas stream entering the cyclone as 10.30 psig (psi gauge). A manometer is also connected between the gas streams entering and leaving the cyclone unit showing a height difference of 20.0 cm . It contains a fluid with a specific gravity of 6.45 . The pressure is expected to drop as the gas flows through the cyclone. If the absolute atmospheric pressure is 100 kPa , what is the pressure of the gas stream leaving the cyclone in absolute pressure in kPa ? [ 15 points]
d) To calculate the centrifugal force on particles in the cyclone, the below equation is used. Show whether the equation is dimensionally consistent. You should know the dimensions, or be able to find the dimensions, for the variables listed. [5 points]

$$
F_{c}=\frac{4}{3} \pi \rho_{p} r_{p}^{3} u_{t}^{2}
$$

$$
\begin{array}{cc}
F_{c}=\text { Centrifugal force } & \rho_{p}=\text { particle density } \\
r_{p}=\text { particle radius } & u_{t}=\text { tangential velocity }
\end{array}
$$

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## Conversion factors

Mass
$1 \mathrm{~kg}=1000 \mathrm{~g}=0.001$ metric ton (tonne) $=2.20462 \mathrm{lb}_{\mathrm{m}}=35.27392 \mathrm{oz}$
$11 \mathrm{~b}_{\mathrm{m}}=16 \mathrm{oz}=5 \times 10^{-4}$ ton $=453.593 \mathrm{~g}=0.453593 \mathrm{~kg}$
Length
$1 \mathrm{~m}=100 \mathrm{~cm}=1000 \mathrm{~mm}=10^{6} \operatorname{microns}(\mu \mathrm{~m})=10^{10}$ Angstroms $(\AA)=39.37 \mathrm{in}=3.2808 \mathrm{ft}=1.0936 \mathrm{yd}$ $=0.0006214 \mathrm{mile}$
$1 \mathrm{ft}=12 \mathrm{in}=1 / 3 \mathrm{yd}=0.3048 \mathrm{~m}=30.48 \mathrm{~cm}$
Volume
$1 \mathrm{~m}^{3}=1000 \mathrm{~L}=10^{6} \mathrm{~cm}^{3}=10^{6} \mathrm{~mL}=35.3145 \mathrm{ft}^{3}=219.97$ imperial gallons $=264.17 \mathrm{gal}=1056.68 \mathrm{qt}$ $1 \mathrm{ft}^{3}=1728 \mathrm{in}^{3}=7.4805 \mathrm{gal}=29.922 \mathrm{qt}=0.028317 \mathrm{~m}^{3}=28.317 \mathrm{~L}$

Density
$1 \mathrm{~g} / \mathrm{cm}^{3}=1000 \mathrm{~kg} / \mathrm{m}^{3}=62.43 \mathrm{lb}_{\mathrm{m}} / \mathrm{ft}^{3}=$ density of liquid water at $4^{\circ} \mathrm{C}$ (for reference specific gravities)
Force

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\(1 \mathrm{~N}=1 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}^{2}=10^{5}\) dynes \(=10^{5} \mathrm{~g} \cdot \mathrm{~cm} / \mathrm{s}^{2}=0.22481 \mathrm{lb}_{\mathrm{f}}\)
\(1 \mathrm{lb}_{\mathrm{f}}=32.174 \mathrm{lb}_{\mathrm{m}} \cdot \mathrm{ft} / \mathrm{s}^{2}=4.4482 \mathrm{~N}=4.4482 \times 10^{5}\) dynes
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Pressure
$1 \mathrm{~atm}=1.01325 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}\left(\right.$ Pa or $\left.\mathrm{kg} / \mathrm{m} \cdot \mathrm{s}^{2}\right)=101.325 \mathrm{kPa}=1.01325 \mathrm{bar}=1.01325 \times 10^{6}$ dynes $/ \mathrm{cm}^{2}$
$=14.696 \mathrm{lb}_{\mathrm{f}} / \mathrm{in}^{2}(\mathrm{psi})=760 \mathrm{mmHg}$ at $0^{\circ} \mathrm{C}($ torr $)=10.333 \mathrm{~m} \mathrm{H}_{2} \mathrm{O}(\mathrm{l})$ at $4^{\circ} \mathrm{C}=29.921$ inches Hg at $0^{\circ} \mathrm{C}$
$=406.8$ inches $\mathrm{H}_{2} \mathrm{O}(\mathrm{l})$ at $4^{\circ} \mathrm{C}$
Energy
$1 \mathrm{~J}=1 \mathrm{~N} \cdot \mathrm{~m}=1 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}^{2}=10^{7} \mathrm{ergs}=10^{7}$ dyne $\cdot \mathrm{cm}=1 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}^{2}=2.778 \times 10^{-7} \mathrm{~kW} \cdot \mathrm{~h}=0.23901 \mathrm{cal}$ $=0.23901 \times 10^{-3} \mathrm{kcal}($ food calorie $)=0.7376 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}}=9.486 \times 10^{-4} \mathrm{Btu}$

Power
$1 \mathrm{~W}=1 \mathrm{~J} / \mathrm{s}=1 \mathrm{~N} \cdot \mathrm{~m} / \mathrm{s}=\mathrm{kg} \cdot \mathrm{m}^{2} / \mathrm{s}^{3}=0.23901 \mathrm{cal} / \mathrm{s}=0.7376 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{s}=9.486 \times 10^{-4} \mathrm{Btu} / \mathrm{s}=1.341 \times 10^{-3} \mathrm{hp}$
Temperature
$\mathrm{T}(\mathrm{K})=\mathrm{T}\left({ }^{\circ} \mathrm{C}\right)+273.15$
$\mathrm{T}\left({ }^{\circ} \mathrm{R}\right)=\mathrm{T}\left({ }^{\circ} \mathrm{F}\right)+459.67$
$\mathrm{T}\left({ }^{\circ} \mathrm{R}\right)=1.8 \cdot \mathrm{~T}(\mathrm{~K})$
$\mathrm{T}\left({ }^{\circ} \mathrm{F}\right)=1.8 \cdot \mathrm{~T}\left({ }^{\circ} \mathrm{C}\right)+32$
Weight equation: $\mathrm{W}=\mathrm{mg}$
At earth's surface: $\mathrm{g}=9.81 \mathrm{~m} / \mathrm{s}^{2}$ or $32.174 \mathrm{ft} / \mathrm{s}^{2}$
$1 \mathrm{~N}=1 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}^{2}$
$1 \mathrm{lb}_{\mathrm{f}}=32.174 \mathrm{lb}_{\mathrm{m}} \cdot \mathrm{ft} / \mathrm{s}^{2}$

Ideal Gas Constant
$\mathrm{R}=8.314 \mathrm{~J} \mathrm{~K}^{-1} \mathrm{~mol}^{-1}=8.314 \mathrm{~L} \mathrm{kPa} \mathrm{K}^{-1} \mathrm{~mol}^{-1}=8.314 \times 10^{-2} \mathrm{~L} \mathrm{bar} \mathrm{K}^{-1} \mathrm{~mol}^{-1}=62.36 \mathrm{~L} \mathrm{Torr} \mathrm{K}^{-1} \mathrm{~mol}^{-1}=1.987 \mathrm{cal}$ $\mathrm{K}^{-1} \mathrm{~mol}^{-1}=0.08206 \mathrm{~L} \mathrm{~atm} \mathrm{~K}^{-1} \mathrm{~mol}^{-1}=10.73 \mathrm{ft}^{3} \mathrm{psi}^{\circ} \mathrm{R}^{-1} \mathrm{lbmol}^{-1}$

Moles
Avogadro's constant $=6.022 \times 10^{23}$ molecules $/ \mathrm{mol}$

## Data Representation and Analysis

Linear Interpolation: $y=y_{1}+\frac{x-x_{1}}{x_{2}-x_{1}}\left(y_{2}-y_{1}\right)$
Linear equations: $y=a x+b ; \quad$ slope $=a=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} ; \quad$ intercept $=b=y_{1}-a x_{1}=y_{2}-a x_{2}$

## Exponent rules

Quotient rule: $\log _{c}(P / Q)=\log _{c}(P)-\log _{c}(Q)$
Power rule: $\log _{c}\left(P^{Q}\right)=Q \log _{c}(P)$
Base switch rule: $\log _{c}(b)=1 / \log _{b}(c)$
Base change rule: $\log _{b}(x)=\log _{c}(x) / \log _{c}(b)$
Other identities: $P=e^{Q} \Leftrightarrow \ln P=Q ; \quad \ln \left(e^{Q}\right)=Q ; \quad e^{\ln P}=P$

## Density

$$
\rho=\frac{m}{V}=\frac{1}{\widehat{V_{m}}} ; \text { Note that } \widehat{V}_{m} \text { is specific volume } \quad S G=\frac{\rho}{\rho_{R E F}}
$$

where $\rho_{\text {REF }}=1 \mathrm{~g} / \mathrm{cm}^{3}$ for solids and liquids (water at $4^{\circ} \mathrm{C}$ ); $\rho_{\text {REF }}=1.2 \mathrm{~kg} / \mathrm{m}^{3}$ for gases (Air at $293 \mathrm{~K}, 1 \mathrm{~atm}$ )
Ideal gas law $-P V=n R T ; \quad$ Molar volume for ideal gasses: $\widehat{V_{n}}=\frac{V}{n}=\frac{R T}{P}$

## Composition

Mole fraction: $x_{i}=\frac{n_{i}}{\sum_{i} n_{i}} ; \quad$ mole percent $=100 \% * x_{i}$
Mass fraction: $w_{i}=\frac{m_{i}}{\sum_{i} m_{i}} ; \quad$ mass percent $=100 \% * \mathrm{w}_{\mathrm{i}}$
Converting between mass and mole fraction: $w_{i}=\frac{x_{i} M_{i}}{\sum_{i} x_{i} M_{i}} \quad x_{i}=\frac{w_{i} / M_{i}}{\sum_{i} w_{i} / M_{i}}$
Molarity $=\left[\right.$ moles $\left./ L_{\text {total }}\right] \quad$ Molality $=\left[\right.$ moles $\left./ \mathrm{kg}_{\text {solvent }}\right]$
By mass (typical for liquids) - Parts per million $(\mathrm{ppm})=\mathrm{w}_{\mathrm{i}} \times 10^{6}=\left(\mathrm{mg}_{\mathrm{i}} / \mathrm{kg}_{\text {total }}\right)$
Parts per billion $(\mathrm{ppb})=\mathrm{w}_{\mathrm{i}} \times 10^{9}=\left(\mu \mathrm{g}_{\mathrm{i}} / \mathrm{kg}_{\text {total }}\right)$

By volume (typical for gases) - Parts per million ( ppm ) $=\mathrm{y}_{\mathrm{i}} \times 10^{6}=\left(\mathrm{mL}_{\mathrm{i}} / \mathrm{m}^{3}{ }_{\text {total }}\right)$
Parts per billion $(\mathrm{ppb})=\mathrm{y}_{\mathrm{i}} \times 10^{9}=\left(\mu \mathrm{L}_{\mathrm{i}} / \mathrm{m}^{3}{ }_{\text {total }}\right)$

Mass/molar ratios - e.g. 1:5 $\mathrm{CH}_{4}: \mathrm{H}_{2} \mathrm{O}$ means 1 methane per 5 waters
Average molecular weight of a mixture - With mole fractions: $\bar{M}=\sum_{i} y_{i} M_{i}$
With mass fractions: $\frac{1}{\bar{M}}=\sum_{i} \frac{w_{i}}{M_{i}}$

## Pressure

Gauge to absolute conversion: $\mathrm{P}_{\text {gauge }}=\mathrm{P}_{\text {absolute }}-\mathrm{P}_{\text {atmospheric }}$
Manometer equation: $\mathrm{P}=P_{o}+\rho g\left(h_{o}-h\right)$

## Solution process

1. Create a list for all assumptions
2. Choose a basis of calculation
3. Draw a flowchart and label all known and unknown variables
4. Express what the problem statement wants in terms of your unknown variables
5. Convert all quantities to the same basis (dimensions)
6. Do a degree-of-freedom analysis
7. Solve equations
8. Check solutions by back substitution
9. Solve additional equations for problem statement
10. Scale basis of calculation

## Equations

General Balance: Input + Generation - Output - Consumption $=$ Accumulation
Degrees of Freedom Analysis: $\mathrm{n}_{\mathrm{df}}=\mathrm{n}_{\text {unknowns }}-\mathrm{n}_{\text {indep eqns }}$

