## EXAM COVER SHEET

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Course code. CHBE 241
Course name: __ Material and Energy Balances
Instructor name: $\qquad$ Jonathan Verrett

Exam duration: $\qquad$
2.5 Hours, 8:30 am to 11:00 am, Dec 7, 2016

## Candidate Information

Name (print, surname first): $\qquad$
Student number: $\qquad$ Signature: $\qquad$

## Permitted Materials

Electronic devices:
Non-programmable calculators
Computers
Course material:
Personal notes

- number of pages: $\qquad$Text book
- sides (e.g. single / dual): $\qquad$Other: $\qquad$
- size (e.g. $81 / 2^{\prime \prime} \times 11$ "): $\qquad$


## Additional Instructions:

- The exam consists of 3 questions and is out of 100 marks
- Make sure to look over all 3 questions and start with those you feel most comfortable with.
- Some parts of questions contain numbers you can use if you could not solve the previous part. Using these is not advantageous compared to using the answer you obtained to the previous part, even if your previous answers turn out to be incorrect.
- You cannot leave the exam in the first 30 minutes (to allow latecomers to arrive) or in the last 15 minutes (to keep an orderly end of the exam).
- You must return all exam material with the exam including the equation sheet, but may detach the equation sheet for ease of use.
- Please have your ID out so we can verify it. When we do we will give you a sticker with your name for your exam booklet
- Best of luck and happy holidays.


## University Rules Governing Formal Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
a. speaking or communicating with other examination candidates, unless otherwise authorized;
b. purposely exposing written papers to the view of other examination candidates or imaging devices;
c. purposely viewing the written papers of other examination candidates;
d. using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
e. using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)—(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

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1. [20 points] The water gas shift reaction is commonly used for producing hydrogen and follows the reaction given below:

$$
\mathrm{CO}+\mathrm{H}_{2} \mathrm{O} \leftrightarrow \mathrm{CO}_{2}+\mathrm{H}_{2}
$$

Say we have a water gas shift reactor with 3 entering streams. The first stream (\#1) contains 10 weight\% $\mathrm{CO}_{2}, 30$ weight\% CO and the remainder as nitrogen ( $\mathrm{N}_{2}$ ) and enters at $100 \mathrm{~kg} / \mathrm{h}$. The second stream (\#2) contains $40 \mathrm{~mol} \%$ hydrogen and the remainder as CO. The third entering stream (\#3) contains only water. Only one product stream exits and it contains a mixture of all the entering species. The reactor achieves an $80 \%$ conversion of the entering CO. The reactants and products enter and leave at $300^{\circ} \mathrm{C}$ and 1 atm and the reactor exchanges energy by giving off or taking in heat to achieve this. The exiting stream is not under equilibrium.

## Physical Data

MW CO $2: 44 \mathrm{~g} / \mathrm{mol}$
MW CO : $28 \mathrm{~g} / \mathrm{mol}$
MW $\mathrm{N}_{2}$ : $28 \mathrm{~g} / \mathrm{mol}$

Perform the following:
a) Draw a flowchart for the process labelling all streams and components. [5 points]
b) Indicate the mole fractions of all components and the total molar flow in kmol/h for stream \#1. [5 points]
c) Do a degrees of freedom analysis on the reactor. Is the problem over-specified, under-specified or adequately specified? [5 points]
d) The pressure gauge for the reactor was found to not be accurate and instead a manometer was installed with one end attached to the reactor and then other end open to the atmosphere. A liquid with a specific gravity of 5.27 (relative to water at $4^{\circ} \mathrm{C}$ ) was used in the manometer. The atmospheric pressure was found to be 100 kPa . The fluid was higher on the side open to the atmosphere by 50 cm . What is the absolute pressure inside the reactor in kPa ? [ 5 points]

## 2. [40 points]

A liquid mixture of $70 \mathrm{~mol} \% \mathrm{n}$-hexane $\left(\mathrm{C}_{6} \mathrm{H}_{14}\right)$ and $30 \mathrm{~mol} \% \mathrm{n}$-heptane $\left(\mathrm{C}_{7} \mathrm{H}_{16}\right)$ at $40^{\circ} \mathrm{C}$ is fed at a rate of $100 \mathrm{~mol} / \mathrm{s}$ into a separating tank, known as a flash drum. In the flash drum, the temperature of the liquid is brought to $80^{\circ} \mathrm{C}$ and leads to the creation of a vapour phase. The vapour and liquid product streams can be assumed to be in equilibrium and are removed continuously at $80^{\circ} \mathrm{C}$. The entire process operates at a uniform 1 atm pressure. Data for this system are provided below:

Table I: Antoine equation coefficients for the $n$-hexane and $n$-heptane, respectively.
The Antoine equation is: $\log _{10} P^{\text {sat }}(\boldsymbol{m m H g})=A-\frac{B}{\boldsymbol{T}\left({ }^{\circ} \mathrm{C}\right)+\boldsymbol{C}}$

| Compound | A | B | C |
| :--- | :--- | :--- | :--- |
| n-hexane | 6.87601 | 1171.17 | 224.41 |
| n-heptane | 6.89677 | 1264.90 | 216.54 |

Table 2: Thermodynamic properties
Note that Hv can be assumed to be independent of temperature and pressure

| Compound | $\mathrm{Cp}[\mathrm{J} /(\mathrm{mol} \mathrm{K})]$ | $\mathrm{Hv}[\mathrm{kJ} / \mathrm{mol}]$ |
| :--- | :--- | :--- |
| n-hexane | 150 | 28.85 |
| n-heptane | 180 | 31.69 |

Perform the following:
a) Calculate the n -hexane and n -heptane molar compositions in the vapour and liquid product streams as well as the product vapour and liquid stream molar flow rates. [25 points]
b) Is energy being transferred to or from the separation tank and what is the amount of energy transferred in kW? [15 points] If you could not solve the previous part (part a), you can use the values that there is $50 \mathrm{~mol} / \mathrm{s}$ of vapour and $50 \mathrm{~mol} / \mathrm{s}$ of liquid produced and that each of these contains $70 \%$ n-hexane and $30 \%$ n-heptane. Do not use these values if you solved part a.
3. [40 points] A furnace is used in a plant to heat a stream of 1 tonne per hour of pure $\mathrm{H}_{2} \mathrm{O}$. The $\mathrm{H}_{2} \mathrm{O}$ enters as a mixture of water (liquid) and steam (vapour) at 5 bar to produce superheated steam at $300^{\circ} \mathrm{C}$ and 5 bar. The heat to do this is provided by the complete combustion of $1 \mathrm{kmol} / \mathrm{h}$ propane $\left(\mathrm{C}_{3} \mathrm{H}_{8}\right)$ with $100 \%$ excess air at 1 atm . The air consists of $79 \mathrm{~mol} \%$ nitrogen $\left(\mathrm{N}_{2}\right)$ and $21 \mathrm{~mol} \%$ oxygen $\left(\mathrm{O}_{2}\right)$ and enters the furnace with the propane at $200^{\circ} \mathrm{C}$. The fractional conversion of propane is 1.00 and the combustion products leave the furnace at $400^{\circ} \mathrm{C}$. The heat of propane combustion forming gaseous water is $-2220 \mathrm{~kJ} /\left(\mathrm{mol} \mathrm{C}_{3} \mathrm{H}_{8}\right)$ at $400^{\circ} \mathrm{C}$. The steam and combustion streams are not in direct contact, but energy is transferred between the two streams and you can assume the furnace to be adiabatic with negligible energy contributions from shaft work, kinetic energy, or potential energy.
$\mathrm{Cp} \_\mathrm{C}_{3} \mathrm{H}_{8}=68 \mathrm{~J} /(\mathrm{mol} \mathrm{K})$
$\mathrm{Cp} \_\mathrm{O}_{2}=29 \mathrm{~J} /(\mathrm{mol} \mathrm{K})$
$\mathrm{Cp} \mathrm{N}_{2}=29 \mathrm{~J} /(\mathrm{mol} \mathrm{K})$ $\mathrm{Cp} \_\mathrm{CO}_{2}=36 \mathrm{~J} /(\mathrm{mol} \mathrm{K})$
$\mathrm{Cp} \mathrm{H}_{2} \mathrm{O}=33.5 \mathrm{~J} /(\mathrm{mol} \mathrm{K})$
Note enthalpy and internal energy values below are in $\mathrm{kJ} / \mathrm{kg}$ and volumes are in $\mathrm{m}^{3} / \mathrm{kg}$
TABLE B. 7 Properties of Superheated Steam ${ }^{a}$

| $\begin{gathered} P(\text { bar }) \\ \left(T_{\text {sat. }}{ }^{\circ} \mathrm{C}\right) \end{gathered}$ |  | Sat'd Water | Sat'd <br> Steam | $\begin{gathered} \text { Temperat } \\ 50 \\ \hline \end{gathered}$ | $\text { C) } \rightarrow$ | 100 | 150 | 200 | 250 | 300 | 350 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 1.0 \\ (99.6) \end{gathered}$ | $\hat{H}$ | 417.5 | 2675.4 | 209.3 | 314.0 | 2676 | 2776 | 2875 | 2975 | 3074 | 3176 |
|  | $\hat{U}$ | 417.5 | 2506.1 | 209.2 | 313.9 | 2507 | 2583 | 2658 | 2734 | 2811 | 2889 |
|  | $\checkmark$ | 0.00104 | 1.69 | 0.00101 | 0.00103 | 1.69 | 1.94 | 2.17 | 2.40 | 2.64 | 2.87 |
| 5.0 | $\hat{H}$ | 640.1 | 2747.5 | 209.7 | 314.3 | 419.4 | 632.2 | 2855 | 2961 | 3065 | 3168 |
| (151.8) | 0 | 639.6 | 2560.2 | 209.2 | 313.8 | 418.8 | 631.6 | 2643 | 2724 | 2803 | 2883 |
|  | $\hat{v}$ | 0.00109 | 0.375 | 0.00101 | 0.00103 | 0.00104 | 0.00109 | 0.425 | 0.474 | 0.522 | 0.571 |
| $\begin{gathered} 10 \\ (179.9) \end{gathered}$ | $\hat{H}$ | 762.6 | 2776.2 | 210.1 | 314.7 | 419.7 | 632.5 | 2827 | 2943 | 3052 | 3159 |
|  | $\hat{0}$ | 761.5 | 2582 | 209.1 | 313.7 | 418.7 | 631.4 | 2621 | 2710 | 2794 | 2876 |
|  | $\hat{\nu}$ | 0.00113 | 0.194 | 0.00101 | 0.00103 | 0.00104 | 0.00109 | 0.206 | 0.233 | 0.258 | 0.282 |

a) Write out the equation for the combustion reaction. What is the molar flow of oxygen and nitrogen into the process? [5 points]
b) What are the molar fractions of all species in the gas exiting the furnace? [10 points]
c) What is the amount of energy transferred due to the combustion reaction in $\mathrm{kJ} / \mathrm{h}$ ? [15 points] If you could not solve the previous section use the following flows, note these are not stoichiometrically correct, but you can use them for the energy balance:

| Substance | Flow in $(\mathrm{kmol} / \mathrm{h})$ | Flow out $(\mathrm{kmol} / \mathrm{h})$ |
| :--- | :--- | :--- |
| C 3 H 8 | 1 | 0 |
| O 2 | 2 | 2 |
| N 2 | 4 | 4 |
| CO 2 | 0 | 6 |
| H 2 O | 0 | 8 |

d) What are the weight fractions of water and steam in the entering pure $\mathrm{H}_{2} \mathrm{O}$ ? [10 points] If you could not solve the previous part, use 100 kW as the energy transferred (you determine whether positive or negative).

## Conversion factors

Mass
$1 \mathrm{~kg}=1000 \mathrm{~g}=0.001$ metric ton (tonne) $=2.20462 \mathrm{lb}_{\mathrm{m}}=35.27392 \mathrm{oz}$
$11 \mathrm{~b}_{\mathrm{m}}=16 \mathrm{oz}=5 \times 10^{-4}$ ton $=453.593 \mathrm{~g}=0.453593 \mathrm{~kg}$
Length
$1 \mathrm{~m}=100 \mathrm{~cm}=1000 \mathrm{~mm}=10^{6}$ microns $(\mu \mathrm{m})=10^{10}$ Angstroms $(\AA)=39.37 \mathrm{in}=3.2808 \mathrm{ft}=1.0936 \mathrm{yd}$ $=0.0006214 \mathrm{mile}$
$1 \mathrm{ft}=12 \mathrm{in}=1 / 3 \mathrm{yd}=0.3048 \mathrm{~m}=30.48 \mathrm{~cm}$
Volume
$1 \mathrm{~m}^{3}=1000 \mathrm{~L}=10^{6} \mathrm{~cm}^{3}=10^{6} \mathrm{~mL}=35.3145 \mathrm{ft}^{3}=219.97$ imperial gallons $=264.17 \mathrm{gal}=1056.68 \mathrm{qt}$ $1 \mathrm{ft}^{3}=1728 \mathrm{in}^{3}=7.4805 \mathrm{gal}=29.922 \mathrm{qt}=0.028317 \mathrm{~m}^{3}=28.317 \mathrm{~L}$

Density
$1 \mathrm{~g} / \mathrm{cm}^{3}=1000 \mathrm{~kg} / \mathrm{m}^{3}=62.43 \mathrm{lb}_{\mathrm{m}} / \mathrm{ft}^{3}=$ density of liquid water at $4^{\circ} \mathrm{C}$ (for reference specific gravities)
Force

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\(1 \mathrm{~N}=1 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}^{2}=10^{5}\) dynes \(=10^{5} \mathrm{~g} \cdot \mathrm{~cm} / \mathrm{s}^{2}=0.22481 \mathrm{lb}_{\mathrm{f}}\)
\(1 \mathrm{lb}_{\mathrm{f}}=32.174 \mathrm{lb}_{\mathrm{m}} \cdot \mathrm{ft} / \mathrm{s}^{2}=4.4482 \mathrm{~N}=4.4482 \times 10^{5}\) dynes
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Pressure
$1 \mathrm{~atm}=1.01325 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}\left(\mathrm{~Pa}\right.$ or $\left.\mathrm{kg} / \mathrm{m} \cdot \mathrm{s}^{2}\right)=101.325 \mathrm{kPa}=1.01325 \mathrm{bar}=1.01325 \times 10^{6}$ dynes $/ \mathrm{cm}^{2}$
$=14.696 \mathrm{lb}_{\mathrm{f}} / \mathrm{in}^{2}(\mathrm{psi})=760 \mathrm{mmHg}$ at $0^{\circ} \mathrm{C}($ torr $)=10.333 \mathrm{~m} \mathrm{H}_{2} \mathrm{O}(1)$ at $4^{\circ} \mathrm{C}=29.921$ inches Hg at $0^{\circ} \mathrm{C}$
$=406.8$ inches $\mathrm{H}_{2} \mathrm{O}(\mathrm{l})$ at $4^{\circ} \mathrm{C}$

## Energy

$1 \mathrm{~J}=1 \mathrm{~N} \cdot \mathrm{~m}=1 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}^{2}=10^{7} \mathrm{ergs}=10^{7}$ dyne $\cdot \mathrm{cm}=1 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}^{2}=2.778 \times 10^{-7} \mathrm{~kW} \cdot \mathrm{~h}=0.23901 \mathrm{cal}$ $=0.23901 \times 10^{-3} \mathrm{kcal}($ food calorie $)=0.7376 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}}=9.486 \times 10^{-4} \mathrm{Btu}$

Power
$1 \mathrm{~W}=1 \mathrm{~J} / \mathrm{s}=1 \mathrm{~N} \cdot \mathrm{~m} / \mathrm{s}=\mathrm{kg} \cdot \mathrm{m}^{2} / \mathrm{s}^{3}=0.23901 \mathrm{cal} / \mathrm{s}=0.7376 \mathrm{ft} \cdot \mathrm{l} \mathrm{b}_{\mathrm{f}} / \mathrm{s}=9.486 \times 10^{-4} \mathrm{Btu} / \mathrm{s}=1.341 \times 10^{-3} \mathrm{hp}$
Temperature
$\mathrm{T}(\mathrm{K})=\mathrm{T}\left({ }^{\circ} \mathrm{C}\right)+273.15$
$\mathrm{T}\left({ }^{\circ} \mathrm{R}\right)=\mathrm{T}\left({ }^{\circ} \mathrm{F}\right)+459.67$
$\mathrm{T}\left({ }^{\circ} \mathrm{R}\right)=1.8 \cdot \mathrm{~T}(\mathrm{~K})$
$\mathrm{T}\left({ }^{\circ} \mathrm{F}\right)=1.8 \cdot \mathrm{~T}\left({ }^{\circ} \mathrm{C}\right)+32$
Weight equation: $\mathrm{W}=\mathrm{mg}$
At earth's surface: $\mathrm{g}=9.81 \mathrm{~m} / \mathrm{s}^{2}$ or $32.174 \mathrm{ft} / \mathrm{s}^{2}$
$1 \mathrm{~N}=1 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}^{2}$
$1 \mathrm{lb}_{\mathrm{f}}=32.174 \mathrm{lb}_{\mathrm{m}} \cdot \mathrm{ft} / \mathrm{s}^{2}$

Ideal Gas Constant
$\mathrm{R}=8.314 \mathrm{~J} \mathrm{~K}^{-1} \mathrm{~mol}^{-1}=8.314 \mathrm{~L} \mathrm{kPa} \mathrm{K}^{-1} \mathrm{~mol}^{-1}=8.314 \times 10^{-2} \mathrm{~L}_{\mathrm{bar} \mathrm{K}}{ }^{-1} \mathrm{~mol}^{-1}=62.36 \mathrm{~L} \mathrm{Torr} \mathrm{K}^{-1} \mathrm{~mol}^{-1}=1.987 \mathrm{cal}$ $\mathrm{K}^{-1} \mathrm{~mol}^{-1}=0.08206 \mathrm{~L} \mathrm{~atm} \mathrm{~K}^{-1} \mathrm{~mol}^{-1}=10.73 \mathrm{ft}^{3} \mathrm{psi}^{\circ} \mathrm{R}^{-1} \mathrm{lbmol}^{-1}$

Moles
Avogadro's constant $=6.022 \times 10^{23} \mathrm{molecules} / \mathrm{mol}$

## Data Representation and Analysis

Linear Interpolation: $y=y_{1}+\frac{x-x_{1}}{x_{2}-x_{1}}\left(y_{2}-y_{1}\right)$
Linear equations: $y=a x+b ; \quad$ slope $=a=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} ; \quad$ intercept $=b=y_{1}-a x_{1}=y_{2}-a x_{2}$

## Exponent rules

Quotient rule: $\log _{c}(P / Q)=\log _{c}(P)-\log _{c}(Q)$
Power rule: $\log _{c}\left(P^{Q}\right)=Q \log _{c}(P)$
Base switch rule: $\log _{c}(b)=1 / \log _{b}(c)$
Base change rule: $\log _{b}(x)=\log _{c}(x) / \log _{c}(b)$
Other identities: $P=e^{Q} \Leftrightarrow \ln P=Q ; \quad \ln \left(e^{Q}\right)=Q ; \quad e^{\ln P}=P$

## Density

$\rho=\frac{m}{V}=\frac{1}{\widehat{V}_{m}} ;$ Note that $\hat{V}_{m}$ is specific volume

$$
S G=\frac{\rho}{\rho_{\text {REF }}}
$$

where $\rho_{\text {REF }}=1 \mathrm{~g} / \mathrm{cm}^{3}$ for solids and liquids (water at $4^{\circ} \mathrm{C}$ ); $\rho_{\text {REF }}=1.2 \mathrm{~kg} / \mathrm{m}^{3}$ for gases (Air at $293 \mathrm{~K}, 1 \mathrm{~atm}$ )
Ideal gas law $-P V=n R T ; \quad$ Molar volume for ideal gasses: $\widehat{V}_{n}=\frac{V}{n}=\frac{R T}{P}$

## Composition

Mole fraction: $x_{i}=\frac{n_{i}}{\sum_{i} n_{i}} ; \quad$ mole percent $=100 \% * \mathrm{x}_{\mathrm{i}}$
Mass fraction: $w_{i}=\frac{m_{i}}{\sum_{i} m_{i}} ; \quad$ mass percent $=100 \% * \mathrm{w}_{\mathrm{i}}$
Converting between mass and mole fraction: $w_{i}=\frac{x_{i} M_{i}}{\sum_{i} x_{i} M_{i}} \quad x_{i}=\frac{w_{i} / M_{i}}{\sum_{i} w_{i} / M_{i}}$
Molarity $=\left[\right.$ moles $\left./ L_{\text {total }}\right] \quad$ Molality $=\left[\right.$ moles $\left./ \mathrm{kg}_{\text {solvent }}\right]$
By mass (typical for liquids) - Parts per million (ppm) $=w_{i} \times 10^{6}=\left(\mathrm{mg}_{\mathrm{i}} / \mathrm{kg}_{\text {total }}\right)$
Parts per billion $(\mathrm{ppb})=\mathrm{w}_{\mathrm{i}} \times 10^{9}=\left(\mu \mathrm{g}_{\mathrm{i}} / \mathrm{kg}_{\text {total }}\right)$
By volume (typical for gases) - Parts per million (ppm) $=y_{i} \times 10^{6}=\left(\mathrm{mL}_{i} / \mathrm{m}_{\text {total }}^{3}\right)$
Parts per billion $(\mathrm{ppb})=\mathrm{y}_{\mathrm{i}} \times 10^{9}=\left(\mu \mathrm{L}_{\mathrm{i}} / \mathrm{m}_{\text {total }}^{3}\right)$
Mass/molar ratios - e.g. 1:5 $\mathrm{CH}_{4}: \mathrm{H}_{2} \mathrm{O}$ means 1 methane per 5 waters
Average molecular weight of a mixture - With mole fractions: $\bar{M}=\sum_{i} y_{i} M_{i}$
With mass fractions: $\frac{1}{\bar{M}}=\sum_{i} \frac{w_{i}}{M_{i}}$

## Pressure

Gauge to absolute conversion: $\mathrm{P}_{\text {gauge }}=\mathrm{P}_{\text {absolute }}-\mathrm{P}_{\text {atmospheric }}$
Manometer equation: $\mathrm{P}=P_{o}+\rho g\left(h_{o}-h\right)$

## Solution process

1. Create a list for all assumptions
2. Choose a basis of calculation
3. Draw a flowchart and label all known and unknown variables
4. Express what the problem statement wants in terms of your unknown variables
5. Convert all quantities to the same basis (dimensions)
6. Do a degree-of-freedom analysis
7. Solve equations
8. Check solutions by back substitution
9. Solve additional equations for problem statement
10. Scale basis of calculation

## Equations

General Balance: Input + Generation - Output - Consumption $=$ Accumulation
Degrees of Freedom Analysis: $\mathrm{n}_{\mathrm{df}}=\mathrm{n}_{\text {unknowns }}-\mathrm{n}_{\text {indep eqns }}$
Fractional excess: fractional excess of $A=\frac{\left(n_{A}\right)_{\text {feed }}-\left(n_{A}\right)_{\text {stoich }}}{\left(n_{A}\right)_{\text {stoich }}}$
Fractional conversion: $f=\frac{\text { moles reacted }}{\text { moles fed }}$ and percent conversion: $\%$ conversion $=f \times 100 \%$
Mole balance on a species: $n_{i}=n_{i 0}+v_{i} \xi$
Equilibrium constant for reaction $\mathrm{aA}+\mathrm{bB} \leftrightarrow \mathrm{cC}+\mathrm{dD}: K(T)=\prod_{i} a_{i, e q}{ }^{v i}=\frac{\left[a_{c, e q}\right]^{c}\left[a_{D, e q}\right]^{d}}{\left[a_{A, e q}\right]^{a}\left[a_{B, e q}\right]^{b}}$
Where activity is: $\quad a_{i} \approx y_{i} P /(1 \mathrm{~atm})$ for gases, $\quad a_{i} \approx x_{i}$ for liquids, $\quad a_{i} \approx 1$ for pure solids
Yield: $\frac{\text { moles of desired product formed }}{\text { moles that would be formed if there were no side reactions }} \times 100 \%$
Selectivity: $\frac{\text { moles of desired product formed }}{\text { moles of the undesired product formed }}$
Overall conversion: $\frac{\text { reactant input to process-reactant output from process }}{\text { reactant input to process }}$
Single-pass conversion: $\frac{\text { reactant input to reactor-reactant output from reactor }}{\text { reactant input to reactor }}$
Stack gasses - Wet basis (includes water), Dry basis does not include water

Theoretical oxygen: moles of O 2 needed for complete combustion of all fuel fed to reactor producing only $\mathrm{CO}_{2}$ (from carbon), $\mathrm{H}_{2} \mathrm{O}$ (from hydrogen) and $\mathrm{SO}_{2}$ (from sulphur).

Theoretical air: quantity of air that contains the theoretical oxygen, air is $21 \%$ oxygen and has a MW of $29 \mathrm{~g} / \mathrm{mol}$
Excess Air/ $\mathrm{O}_{2}$ : amount by which the air/ $\mathrm{O}_{2}$ fed into the reactor exceeds the theoretical air $/ \mathrm{O}_{2}$.
Percent excess air: $\frac{(\text { moles air } / O 2)_{\text {fed }}-(\text { moles air } / O 2)_{\text {theoretical }}}{(\text { moles air } / O 2)_{\text {theoretical }}} \times 100 \%$

## Degrees of Freedom Calculations

## Molecular Species DOF

+ \# unknowns
+ \# independent chemical reactions
- \# independent species balances
- \# other equations relating unknowns
$=$ DOF


## Atomic Species DOF

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+ \# unknowns
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- \# independent atomic species balances
- \# independent molecular species balances on non-reacting species (includes inerts)
- \# other equations relating unknowns
$=$ DOF


## Reaction Extent DOF

+ \# unknowns
+ \# independent reactions (1 unknown $\xi$ for each reaction)
- \# independent reactive species (1 equation with $\xi$ involved for each reactive species)
- \# independent molecular species balances on non-reacting species (includes inerts)
- \# other equations relating unknowns
= DOF
Special case: splitters - all material balances reduce to 1 independent equation


## Chapter 5/6 - Phase Equilibrium

Density of Mixtures: by volume additivity $\frac{1}{\bar{\rho}}=\sum_{i=1}^{n} \frac{w_{i}}{\rho_{i}} \quad$ by simple average $\bar{\rho}=\sum_{i=1}^{n} w_{i} \rho_{i}$
Ideal Gas Law: $\quad P V=n R T \quad$ or $\quad P \dot{V}=\dot{n} R T \quad$ or $\quad P \hat{V}=R T$
Error on the ideal gas equation is less than $1 \%$ if: $\quad \hat{V}_{\text {ideal }}=\frac{R T}{P}>\frac{5 L}{m o l}$. or $\frac{80 f t^{3}}{l b m o l}$ (diatomic gasses)

$$
\widehat{V}_{\text {ideal }}=\frac{R T}{P}>\frac{20 L}{m o l} . \text { or } \frac{320 f t^{3}}{\text { lbmol }}(\text { other gasses })
$$

Standard Temperature $\left(0^{\circ} \mathrm{C}\right)$ and Pressure $(1 \mathrm{~atm}): \hat{V}_{S}=22.4 \frac{\mathrm{~m}^{3}(S T P)}{\text { kmol }}=22.4 \frac{L(S T P)}{\mathrm{mol}}=359 \frac{f t^{3}(S T P)}{\mathrm{lbmol}}$
Ideal gas mixtures

$$
\begin{array}{ll}
\text { Partial pressure: } p_{A}=y_{A} P & \text { and Dalton's Law: } P=\sum_{i=1}^{n} p_{i}=p_{A}+p_{B}+\cdots \\
\text { Partial volume: } v_{A}=y_{A} V & \text { and Amgat's Law: } V=\sum_{i=1}^{n} v_{i}=v_{A}+v_{B}+\cdots
\end{array}
$$

Reduced Properties Temperature: $T_{r}=T / T_{c} \quad$ Pressure:: $P_{r}=P / P_{c}$
Virial equation of state: $\frac{P \widehat{V}}{R T}=1+\frac{B}{\widehat{V}}+\frac{C}{\widehat{V}^{2}}+\frac{D}{\widehat{V}^{3}}+\cdots$
1-term approximation of Virial equation of state: $\frac{P \widehat{V}}{R T}=1+\frac{B P}{R T} \quad$ where $B=\frac{R T_{c}}{P_{c}}\left(B_{0}-\omega B_{1}\right)$
$B_{0}=0.083-\frac{0.422}{T_{r}^{1.6}} \quad B_{1}=0.139-\frac{0.172}{T_{r}^{4.2}}$
Van der Waals equation of state: $P=\frac{R T}{\hat{V}-b}-\frac{a}{\hat{V}^{2}} \quad$ where: $a=\frac{27 R^{2} T_{c}^{2}}{64 P_{c}} \quad b=\frac{R T_{c}}{8 P_{c}}$
Compressibility factor equation of state: $P V=z n R T \quad$ or $\quad P \widehat{V}=z R T$
Gibb's phase rule: $D F=2+c-\Pi-r$

- DF [=] degrees of freedom (\# intensive variables that can be specified)
- c [=] number of independent chemical species
- $\Pi[=]$ number of phases in a system at equilibrium
- $\mathrm{r}[=]$ number of independent equilibrium chemical reactions among the species


## Vapour-Liquid Equilibrium

Clapeyron equation $\frac{d p^{*}}{d T}=\frac{\Delta \hat{H}_{v}}{T\left(\widehat{V}_{g}-\widehat{V}_{l}\right)}$
Clausius-Clapeyron eqn: $\frac{d\left(\ln p^{*}\right)}{d(1 / T)}=\frac{\Delta \widehat{H}_{v}}{R}$ Assuming P is low so that $\widehat{V}_{g}-\widehat{V}_{l}=\widehat{V}_{g}$ and $\Delta \widehat{H}_{v}$ independent of T
Integrating, we get: $\ln p^{*}=\frac{-\Delta \hat{H}_{v}}{R T}+B \quad$ or $\quad \ln \left(p_{1}^{*} / p_{2}^{*}\right)=\frac{-\Delta \hat{H}_{v}}{R}\left(\frac{1}{T_{1}}-\frac{1}{T_{2}}\right)$
Antoine eqn: $\log _{10} p^{*}=A-\frac{B}{T+C}$

Raoult's Law: $p_{i}=y_{i} \times P=x_{i} \times p_{i}^{*}(T)$ with $p_{i}^{*}(T)$ meaning vapour pressure [ $p_{i}^{*}$ ] is a function of temp. [ T$]$
Henry's Law: $p_{i}=y_{i} \times P=x_{i} / H_{i}(T)$
With $H_{i}(T)$ is the Henry's Law constant for species i in a specific solvent and is a function of temperature [T]

Relative Saturation (Relative Humidity): $s_{r}\left(h_{r}\right)=\frac{p_{i}}{p_{i}^{*}(T)} \times 100 \%$
Molal Saturation (Molal Humidity): $s_{m}\left(h_{m}\right)=\frac{p_{i}}{P-p_{i}}=\frac{\text { moles of vapour }}{\text { moles of "vapour free" (dry)gas }}$
Absolute Saturation (Absolute Humidity): $s_{a}\left(h_{a}\right)=\frac{p_{i} M_{i}}{\left(P-p_{i}\right) M_{d r y}}=\frac{\text { mass of vapour }}{\text { mass of dry gas }}$
Percentage Saturation (Percentage Humidity): $s_{p}\left(h_{p}\right)=\frac{s_{m}}{s_{m}^{*}} \times 100 \%=\frac{p_{i} /\left(P-p_{i}\right)}{p_{i}^{*} /\left(P-p_{i}^{*}\right)} \times 100 \%$
Bubble point temperature: $P=\sum_{i=1}^{n} x_{i} p_{i}^{*}\left(T_{B P}\right)=x_{A} p_{A}^{*}\left(T_{B P}\right)+x_{B} p_{B}^{*}\left(T_{B P}\right)+\cdots$
Bubble point pressure: $P_{B P}=1 / \sum_{i=1}^{n} \frac{y_{i}}{p_{i}^{*}\left(T_{B P}\right)}$
Dew point temperature: $P=\sum_{i=1}^{n} x_{i} p_{i}^{*}\left(T_{D P}\right)=x_{A} p_{A}^{*}\left(T_{D P}\right)+x_{B} p_{B}^{*}\left(T_{D P}\right)+\cdots$
Dew point pressure: $P_{D P}=1 / \sum_{i=1}^{n} \frac{y_{i}}{p_{i}^{*}\left(T_{D P}\right)}$

## Colligative Properties

Vapour pressure lowering: $\Delta p_{s}^{*}=p_{s}^{*}-\left(p_{s}^{*}\right) e=x \times p_{s}^{*}$
Boiling point elevation: $\Delta T_{b}=T_{b s}-T_{b 0}=\frac{R\left(T_{b 0}\right)^{2}}{\Delta \widehat{H}_{v}} x$
Freezing point depression: $\Delta T_{m}=T_{m 0}-T_{m s}=\frac{R\left(T_{m 0}\right)^{2}}{\Delta \hat{H}_{m}} x$

Liquid-Liquid Distribution coefficient or partition ratio: $K=\frac{\left(w_{A}\right)_{\text {phase } 1}}{\left(w_{A}\right)_{\text {phase } 2}}$

## Adsorption

Linear region: $X_{i}^{*}=K c_{i} \quad$ or $\quad X_{i}^{*}=K^{\prime} p_{i} \quad$ Where $\mathrm{X}^{*}$ is in g adsorbate/g adsorbent
Langmuir Isotherm: $X_{i}^{*}=\frac{a K_{L} c_{i}}{1+K_{L} c_{i}} \quad$ or $\quad X_{i}^{*}=\frac{a K_{L}{ }^{\prime} p_{i}}{1+K_{L} p_{i}} \quad$ Where a, K's are fitted to data

## Chapter 7-9 Energy Balances

Closed systems: $\Delta U+\Delta E_{k}+\Delta E_{p}=\mathrm{Q}+\mathrm{W} \quad$ Where: $\Delta U=U_{f}-U_{i}$
Open systems: $\Delta \dot{H}+\Delta \dot{E}_{k}+\Delta \dot{E}_{p}=\dot{Q}+\dot{W}_{s} \quad$ Where: $\Delta \dot{H}=\Delta \dot{U}+\dot{W}_{f l}=\Delta \dot{U}+\Delta(P \dot{V})$
Note: Adiabatic in an open or closed system means Q or $\dot{Q}=0$
Energy changes due to changes in pressure: For solids and liquids: $\Delta \widehat{H}=\Delta \widehat{U}+\Delta(P \widehat{V}) \approx \widehat{V} \Delta P$, all others $=0$ Energy changes due to changes in temperature:

Closed systems: $\Delta \widehat{U}=\int_{T_{1}}^{T_{2}} C_{v}(T) d T$
Liquids and Solids: $C_{p} \approx C_{v}$

Open systems: $\Delta \widehat{H}=\int_{T_{1}}^{T_{2}} C_{p}(T) d T$
Ideal Gases: $C_{p} \approx C_{v}+R$

## Energy changes due to phase changes:

Open systems: $\Delta \widehat{H}_{m}(T)$ or $\Delta \widehat{H}_{v}(T)=$ heat of melting or vapourization at a specific temperature ( T )
Closed systems: For solids/liquid: $\Delta \widehat{U}_{m} \approx \Delta \widehat{H}_{m} \quad$ For ideal gases: $\Delta \widehat{U}_{v} \approx \Delta \widehat{H}_{v}-R T$

## Estimating heats of vapourization:

Note: $T_{b}$ is the normal boiling point temperature ( 1 atm ), $T_{c} / P_{c}$ are critical temperature and pressure
Trouton's rule
Nonpolar liquids: $\Delta \widehat{H}_{v}(k J / m o l)=0.088 T_{b}(K)$ Water, Low MW alcohols: $\Delta \widehat{H}_{v}(k J / m o l)=0.109 T_{b}(K)$
Chen's equation for Nonpolar liquids: $\Delta \widehat{H}_{v}(\mathrm{~kJ} / \mathrm{mol})=\frac{T_{b}\left[0.0331\left(T_{b} / T_{c}\right)-0.0327+0.0297 \log _{10} P_{c}\right]}{1.07-\left(T_{b} / T_{c}\right)}$
Watson correlation: $\Delta \widehat{H}_{v}\left(T_{2}\right)=\Delta \widehat{H}_{v}\left(T_{1}\right)\left(\frac{T_{c}-T_{2}}{T_{c}-T_{1}}\right)^{0.38}$

## Psychrometric charts

Wet bulb temperature - temperature shown by a thermometer with a moist wick around its bulb

## Energy changes due to phase changes:

Heat of solution $\Delta \widehat{H}_{s}(T, r)$ - change in enthalpy when 1 mole of solute is dissolved in " $r$ " moles of solvent at a given temperature " $T$ "

## Reactive Energy Balances

Heat of reaction at a given T, P:

$$
\Delta H_{r}(T, P)=H_{\text {products }}-H_{\text {reactants }}
$$

$\Delta H_{r}>0$ endothermic (positive)
$\Delta H_{r}<0$ exothermic (negative)
Heat of formation $\left(\Delta \boldsymbol{H}_{\boldsymbol{f}}\right)$ of a compound is found from the combination of its elemental components as found naturally (eg. $\mathrm{O}_{2}$ )

Standard heat of combustion $\left(\Delta \widehat{\boldsymbol{H}}_{c}^{o}\right)$ - Heat of reaction of an amount of the given compound with oxygen to yield specified products $\left(\mathrm{CO}_{2}, \mathrm{H}_{2} \mathrm{O}, \mathrm{SO}_{2}, \mathrm{~N}_{2}\right)$ where reactants and products are at $25^{\circ} \mathrm{C}, 1 \mathrm{~atm}$.

## Heat of Reaction:

From Heats of Formation: $\Delta \boldsymbol{H}_{r}^{o}=\sum \boldsymbol{v}_{\boldsymbol{i}} \cdot \Delta \widehat{\boldsymbol{H}}_{f, i}^{o}=\sum_{\text {products }}\left|\boldsymbol{v}_{i}\right| \cdot \Delta \widehat{\boldsymbol{H}}_{f, i}^{o}-\sum_{\text {reactants }}\left|\boldsymbol{v}_{i}\right| \cdot \Delta \widehat{\boldsymbol{H}}_{f, i}^{o}$
From Heats of Combustion: $\Delta \boldsymbol{H}_{r}^{o}=-\sum \boldsymbol{v}_{\boldsymbol{i}} \cdot \Delta \widehat{\boldsymbol{H}}_{c, i}^{o}=\sum_{\text {reactants }}\left|\boldsymbol{v}_{i}\right| \cdot \Delta \widehat{\boldsymbol{H}}_{c, i}^{o}-\sum_{\text {products }}\left|\boldsymbol{v}_{i}\right| \cdot \Delta \widehat{\boldsymbol{H}}_{c, i}^{o}$

## Solution process for non-reactive energy balance problems

1. Create a list for all assumptions
2. Draw a flowchart and label all known and unknown variables
3. Do a degree-of-freedom analysis (if required)
4. Solve material balances as much as possible
5. Select appropriate reference states based on thermodynamic data available
6. Create inlet and outlet table with enthalpies
7. Solve energy balance

## Solution strategies for energy balances on reactive processes

Heat of Reaction Method - Generally preferable for a single reaction system where $\Delta H r$ is known.

1. Same as non-reactive method, except
2. Calculate the extent of each reaction: $\xi=\frac{\left|\left(\dot{n}_{i}\right)_{o u t}-\left(\dot{n}_{i}\right)_{\text {in }}\right|}{\left|\left(\dot{v}_{i}\right)\right|}$
3. Calculate $\Delta \dot{H}$ using $\Delta \dot{H}=\sum_{\text {reactions }} \xi \Delta \dot{H}_{r}^{o}+\sum \dot{n}_{\text {out }} \cdot \widehat{H}_{\text {out }}-\sum \dot{n}_{\text {in }} \cdot \widehat{H}_{\text {in }}$ taking into account temperature or phase changes with $\widehat{H}$ 's

Heat of Formation Method - Generally preferable for a multiple reaction system where $\Delta H r$ is unknown.

1. Same as non-reactive method, except
2. Find enthalpies of compounds based on heat of formation, taking into account temperature or phase changes, e.g. $\widehat{H}_{C 3 H 8(g)}=\Delta \widehat{H}_{f, C 3 H 8(l)}^{o}+\int_{T_{r e f}}^{T_{b}} C_{p, C 3 H 8(l)} \mathrm{dT}+\Delta \widehat{H}_{v, C 3 H 8}\left(T_{b}\right)+\int_{T_{b}}^{T} C_{p, C 3 H 8(v)} \mathrm{dT}$
3. Calculate $\Delta \dot{H}$ using $\Delta \dot{H}=\sum \dot{n}_{\text {out }} \cdot \widehat{H}_{\text {out }}-\sum \dot{n}_{\text {in }} \cdot \widehat{H}_{\text {in }}$

## Fuel Properties

$\mathrm{HHV}=$ Higher heating value (water produced as a liquid) and $\mathrm{LHV}=$ lower heating values (water as a vapour):

$$
H H V=L H V+n_{H_{2} O} * \Delta \widehat{H}_{v}\left(H_{2} O, T\right)
$$

Adiabatic Flame Temperature - temperature of exit gas if all energy from combustion goes into heating the gas:

$$
\Delta H=n_{\text {fuel }} * \Delta \widehat{H}_{c}^{o}+\sum_{\text {out }} n_{i} \widehat{H}_{i}\left(T_{\text {ad }}\right)-\sum_{\text {in }} n_{i} \widehat{H}_{i}\left(T_{i 0}\right)=0
$$

Lean - $\mathrm{O}_{2}$ in excess, with a certain limit where there is so much air in the fuel-air mixture that the fuel cannot explode (called the lower flammability limit)

Rich - fuel in excess, with a certain limit where there is so much fuel in the fuel-air mixture that the fuel cannot explode (called the upper flammability limit)

The flash point is the temperature at which a pure liquid fuel gives off enough vapour to form an ignitable mixture with air above the liquid surface.

The autoignition temperature is the temperature where enough energy is provided by the reaction to heat incoming reactants

