MATH 600D: ASSIGNMENT 3

DUE: OCT 25, 2011

(1) Let R be a ring and P a finitely generated R-module. Show that there is a well-defined homomorphism $Aut(P) \to K_1(R)$. Use this to show there is a natural product operation $K_0(R) \otimes K_1(S) \to K_1(R \otimes S)$.

(2) Show that the rings R and $M_n(R)$ are **Morita** equivalent and conclude that $K_i(R) = K_i(M_n(R))$ for $i = 0, 1$.
(Two rings R and S are said to be Morita equivalent if the categories R-mod and S-mod are equivalent.)

(3) Let (r, s) be a unimodular row over a commutative ring R. Define *Mennicke symbol* $\begin{bmatrix} r \\ s \end{bmatrix}$ to be the class in $SK_1(R)$ of the matrix $\begin{pmatrix} r & s \\ t & u \end{pmatrix}$ where $t, u \in R$ satisfy $ru - st = 1$. Show that this symbol is independent of the choice of t and u, and that we have

(a) $\begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} s \\ r \end{bmatrix}$

(b) $\begin{bmatrix} s \\ r \end{bmatrix} \begin{bmatrix} s' \\ r \end{bmatrix} = \begin{bmatrix} ss' \\ r \end{bmatrix}$

(4) Consider the functions $\rho_n : R^{n-1} \to St_n(R)$ sending $(r_1, \cdots, r_{n-1})$ to the product homomorphism $x_{in}(r_1)x_{2n}(r_2)\cdots x_{n-1,n}(r_{n-1})$. Show using Steinberg relations that this is a group homomorphism. Show that ρ_n is an injection by showing that the composition $\phi \circ \rho : R^{n-1} \xrightarrow{\rho} St_n(R) \to GL_n(R)$ is an injection.